MATH 300 Introduction to Advanced Mathematics SYLLABUS

Prof. Sachs Spring 2021

TEXT: No printed textbook required. Notes from me along with access to an open source book for some initial material will be the "textbook". A link to the online book is on Blackboard. My book manuscript will be forthcoming on Blackboard also.

COURSE OVERVIEW: This course aims to prepare students for success in later courses in advanced undergraduate mathematics and beyond. "Mathematical maturity" includes clear reasoning, writing and speaking about mathematics, proving results, but also building skills in problem solving, investigation, conjecture, visualization, computing, and making connections to prior knowledge. The processes of abstraction and generalization will be explicitly acknowledged during the course. A range of proof techniques will appear and we will discuss the thinking behind these techniques. The mathematics we learn should enhance your interest in the subject also.

The central theme of our experimental variant is the complex number system. The complex setting is simultaneously similar to and yet quite different from corresponding real number situations. Building on some high school experiences and calculus of one real variable, we will investigate specific examples and develop general statements about complex polynomials, rational functions, and some of the transcendental functions familiar in calculus for a real input. There will be glimpses of algebra, analysis, geometry, number theory, combinatorics, and topology in our work, along with a dose of history. Our visualizations will be beautiful but will require interpretive skills also. Historically the development of these topics led to many branches of modern mathematics emerging or evolving and are fundamental to modern physics and signal processing.

WARNING: We are experimenting with this course content. The proposed topics and schedule is definitely subject to change based on your feedback.

MEETING: Mon. and Weds. 3:00 – 4:15 pm in Blackboard Collaborate

OPEN STUDENT HOURS: Monday 4:30-5:30pm Thursday (online) and by appt.

CONTACT INFO: E-MAIL: rsachs@gmu.edu
Use headers to avoid spam filtering!

LA: Swan Klein (hklein2@masonlive.gmu.edu)
Swan's open times for help: 4-5pm Tuesday and 1-2pm Friday online.

COURSE WEB PAGE: Blackboard page at mymasonportal

GRADING: The grading scale is as follows, and is based on your correctly rounded semester average. There will likely be no curve. A+: 98+ A: 93 - 97; A-: 90 - 92; B+: 88 - 89; B: 83 - 87;

B-: 80 - 82; C+: 78 - 79; C: 73 - 77; C-: 70 - 72; D: 60 - 69; F: 0 - 59 Grading will be fair and impartial. Points used as the basis of the grade will be: Hmwk. (400 pts.); Class Write-ups (250 pts.); Midterm Exam (150 pts.); Final exam (200 pts.); Longer Writing Assignments (400 pts.)

Writing Assignments: There will be approximately 10 short writing assignments given throughout the semester, along with two lengthier essays that are more expository than the short ones. The first of these will require a revised submission after feedback on the initial version. The short assignments will involve writing mathematically and grammatically correct solutions to problems, usually involving proofs. I am requesting you use LaTeX to do this. We will help you learn the basics of this and encourage your use of overleaf also. Your grade for these assignments will be based on multiple criteria: the correctness of your proofs for designated problems and clarity and correctness of your writing in general, along with effort even if unsuccessful on more challenging problems. Your writing assignments which will total at least 3500 words according to the guidelines of the Writing Across the Curriculum Committee. Feedback will be given. Precise assignments and due dates will given on Blackboard. Collaboration is not permitted on these assignments.

Collaborative Work: Most class periods there will be assigned several problems whose solution will take the form of collaborative work done primarily in class. The work will be done in groups of no more than three that either I will assign or that you will choose. Some class periods there will be a new assignment given and in others you will be asked to finish the assignment given out previously. You will be asked to write up the solutions you come up with as a group and submit them. These submissions will be subject to the same requirements as the homework assignments in the previous item and will be graded in the same way. Your score on these assignments will comprise 250 points.

OTHER POLICIES: The GMU Honor code is in effect at all times and students are expected to be fully aware of its requirements. Group work may be part of the course, in which case group members will truthfully report on non-contributing members. Absence from quizzes and exams must be for a valid reason and requires prior notification except in extreme circumstances. DO NOT ARRANGE TO LEAVE BEFORE THE FINAL EXAM. If you are a student with a disability and you need academic accommodations, please see me and contact the Office of Disability Resources at 703/993-2474. All academic accommodations must be arranged through that office.

EXAMS: Midterm Exam **Tentative** Monday, March 9
Final Exam **Definitely** 5/3 Main slot is 1:30 pm – 4:15 pm

See the Registrar page at https://registrar.gmu.edu/calendars/spring-2021/ for deadlines.

MATERIAL COVERED AND TENTATIVE WEEKLY SCHEDULE This schedule is likely too rapid so we will adjust as needed. So much wonderful material here!

- Week 1: Overview of course; basics of complex numbers; the complex plane – coordinates, vector view, polar coordinate view. Complex conjugation. Modulus of complex number. Linking properties of complex numbers to real numbers and/or vectors in 2-D. What are the Gaussian integers? Writing discussion part 1: some basics of mathematical writing – use of symbols, quantifiers, clarity, connecting equations for the reader.

- Week 2: Complex multiplication and its algebra and geometry continued; complex linear functions; linear functions as algebraic objects; squaring and square roots; quadratic formula revisited; integer powers of z as functions and solving $z^n = w$ for z given w. How roots are related for powers. Roots of unity introduced and studied.

- Week 3: Polynomials – definitions; some basic properties; multiplying polynomials; division and the Euclidean algorithm; roots and factors; GCD of two polynomials; division and idea of complex derivative for polynomials. Writing discussion part 2: brevity balanced with clarity – the case of the world's shortest proof of a serious theorem. How to untangle dense writing.

- Week 4: Cardano's formula for cubics; Bombelli's strategy; solving quartics and the dream of solving all polynomials by radicals. Viete and symmetric polynomials in roots. Cardano's formula revisited in light of symmetric polynomials in roots. Local view of polynomial functions. Thinking about Fundamental Theorem of Algebra.

- Week 5: Rational functions; equivalence classes used in for common factors; algebraic view as a field; the special case of rational functions of degree 1; local view of rational functions. Writing discussion part 3: the art of showing appropriate level of detail.

- Week 6: Euler's formula revisited; the complex exponential function from one viewpoint. Properties that are similar in complex setting and those that are different. Solving for $w = e^z$ given w.

- Week 7: Mid-term Exam; series viewpoint of exponential and the need for derivatives.

- Week 8: Calculus of one variable revisited in the complex plane – basic definitions of limits, continuity, and derivative.

- Week 9: The new meaning of derivatives; geometry of complex differentiable mappings. Some added perspective on Fundamental Theorem of Algebra. Writing discussion part 4: absorbing critiques including your own and how to revise well.

- Week 10: Rational function of degree 1 revisited. Geometric properties. Infinity. Projective view using homogeneous coordinates. The cross-ratio as an invariant.

- Week 11: Power series revisited using complex numbers; real power series and radius of convergence tied to complex singularities. Writing discussion part 5: technical writing versus expository writing in math.

- Week 12: Introduction to Fourier series: the recipe for coefficients; issues of convergence mentioned; formal version of Parseval; integration by parts and smoothness connected to decay of Fourier coefficients.
- Week 13: The finite Fourier transform. Discrete analogy; justification of Parseval using limits; alternative view of this as interpolation for polynomials and as algebraic evaluations on polynomials. Writing discussion part 6: open for student questions and input.
- Week 14: Some non-Euclidean geometry - upper half plane model of Poincare and its isometries; Link to special relativity (time permitting); idea of a Riemann surface (time permitting).

Writing in the Math Major: This course has been approved by the Faculty Senate Writing Across the Curriculum Committee to fulfill all of the Writing Intensive requirement in the Mathematics majors. It does so through a 1000-word paper due in two stages (revision after comments), the many homework and in class problem solution write-ups, and the 2000-word final expository paper. The first expository paper will be completed through a draft/feedback/revision process. The first draft will be submitted, I will provide commentary on the draft, and the revised draft will be three weeks after you receive the comments.

Mathematical Communication: Your communication of mathematics and your thinking will be developed in many modes besides writing. Speaking and collaborating will enhance your understanding and provide a valuable life skill. We will use videos and animations at times, though not asking you to create them. Sketches and diagrams are part of typical mathematical communication.