George Mason University
Department of Mathematical Sciences

Algebraic Combinatorics

Spring 2021

Course: MATH-629, Topics in Algebra, section 001.

Total Credits: 3.

Purpose: This topics course will serve as an introduction to the application of commutative algebra in combinatorics and as a possible bridge between these two hitherto independent topics; commutative algebra (including rings, modules and fields) and combinatorics (including graph theory, enumeration and discrete geometry). We will use abstract algebra as an important tool to tackle and solve some combinatorial problems. Since the fundamental purpose will be to obtain combinatorial results, the course is called "*algebraic combinatorics*", but it might as well be called "*commutative algebra in combinatorics*" if one prefers to put more weight on the algebraic tools themselves, instead of the final combinatorial interpretation. – It will hopefully be demonstrated that algebraic techniques are powerful and indispensable in numerous combinatorial investigations.

The hope is that, after this course, it should be clear that a firm knowledge in commutative algebra is necessary for the working combinatorist, just as it is for many other branches of mathematics like functional analysis and algebraic topology. Finally, we hopefully will have covered enough topics in algebra and combinatorial applications for the attenders to be able to read advanced monographs and some research articles on algebraic combinatorics, related topics and a bit beyond.

Prerequisites: Most important is to have mathematical maturity and an open mind. Roughly what I expect is listed as follows:

- Required – Math 621, Algebra (First year graduate course), or equivalent.
- Preferred – Math 325, Discrete Mathematics II, or equivalent.

NOTE! This is more specific than what is stated in the official WebSite: `catalog.gmu.edu/` (Click on "Course" and write "math 629".)

Times and Places: online, asynchronous instructional method.

Period: From January 25. to May 10.

Professor:
Geir Agnarsson
email: `gagnarss@gmu.edu`

Office-hours: W 1 – 3 pm via Zoom.

Course Text: The lectures will be based on lecture notes and there is no required text. The lectures will be taken from several books, monographs and perhaps some research articles, and will be self contained.

Below is a list of the most relevant books, from some of which most of the lectures will be taken, and it serves as the main list of references.

1. Takayuki Hibi, *Algebraic Combinatorics; on Convex Polytopes*, Carslaw Publications, (1992).

2. Richard P. Stanley, *Combinatorics and Commutative Algebra*, 2nd Edition, Progress in Mathematics (PM-41), Birkhäuser, Boston, (1996).

3. Ezra Miller; Bernd Sturmfels, *Combinatorial Commutative Algebra*, Graduate Texts in Mathematics (GTM-227), Springer Verlag, New York, (2005).

4. Arne Brøndsted, *An Introduction to Convex Polytopes*, Graduate Texts in Mathematics (GTM-90), Springer Verlag, New York, (1983).

Material: Selected topics on the following: (i) convex polytopes, (ii) simplicial complexes, (iii) f-vectors and h-vectors, (iv) Cohen-Macaulay rings, (v) Hilbert functions and Hilbert series, (vi) Stanley-Reisner rings (aka face rings), (vii) Cohen-Macaulay complexes, (viii) Shellability.

Homework: Homework will be assigned sporadically and not on a regular basis. Expect just a few assignments during the semester (3 – 4, or about once a month) to serve as a jolt to read the lecture notes, watch the recordings and participate.

Blackboard: All announcements, notes and pdf file handouts for this course will be posted on Blackboard for this course: MATH 629-001.

Examinations: There will be no exams in this course.

Grading: Your grade for this course will be based on participation and homework.

Geir Agnarsson
January 25, 2021