MATH 639: GEOMETRIC GROUP THEORY
SPRING 2020

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1. Syllabus

1.1. **Overview.** Geometric group theory studies groups using geometry, often by zooming out on relevant graphs or manifolds in order to use analytic techniques. This approach bridges algebra and analysis using geometry, using methods and providing results in all three fields, and has been extremely fruitful starting with Klein's 1800s Erlangen program and continuing with recent work spearheaded by Gromov and Thurston.

In this survey class, we will take a tour of the topics of geometric group theory. We will start with computational questions about abstract groups given by presentations and relations. We will then observe groups as symmetries of geometric objects including certain graphs, Lie groups, and hyperbolic spaces. This will introduce some local choices in the definitions that we will avoid by zooming out to infinity, leading to new intrinsically-fractal spaces where analysis can come into play. While these spaces are not intrinsically-Euclidean, we will see how useful notions of geometry and analysis can be built up on many of them, and how these spaces also arise outside of geometric group theory.

No prerequisites will be strictly necessary, beyond excitement to see a wide swath of mathematics, as we will quickly discuss all necessary background material as it comes up. In addition to important milestones (e.g. the Mostow Rigidity Theorem and the Banach-Tarksi Paradox), we will discuss famous open conjectures and current research directions.

1.2. **Logistics.** The class will meet on Thursdays 7:20-10pm, in 4106 Exploratory Hall. Given the late time slot, students are encouraged to bring snacks. Office hours are by available by request.

The class will be primarily lecture/discussion-based. Students are expected to contribute to the conversation, ask lots of questions, and initiate discussions.

Students will be asked to provide a 20-minute presentation on the last day of the course, based on a PhD thesis of their choice. In preparation for this, each student will submit the following reports (2-pages in standard LaTeX format):

Date: June 15, 2020.

(1) (March 19) What topic in GGT or related fields strikes you as interesting? Who are 3 active researchers in the field, and what are they working on? Name their recently-graduated students.
(2) (April 9) What 3 theses are you considering reporting on? Provide a quick description of each one, and attach a PDF of the thesis. Note that short theses are generally denser and harder to make sense of.
(3) (April 30) Notes for your talk, to be shared with others in the class. With your permission, these will be appended to the course notes.

Grades will be based on attendance and participation. Students who are unable to attend the last session of the course will be required to submit a video presentation instead, to be shared with the class.

1.3. **Literature.** Lectures will draw on a variety of sources. The books include, but are not limited to:

(1) *Office Hours with a Geometric Group Theorist*, ed. Clay and Margalit
(2) *Word Processing in Groups*, ed. Epstein
(3) *Discrete Subgroups of Lie Groups*, Raghanathan
(4) *Metric Spaces of Non-Positive Curvature*, Bridson and Haefliger
(5) *Hyperbolic Groups*, Gromov
(6) *Three-dimensional Geometry and Topology*, Thurston
(7) *Topology*, Munkres