Linear Analysis I

Math 675-001

Fall 2019

This is the web page **http://math.cos.gmu.edu/~wanner/courses/m675f19/index.html**
It will be updated regularly and always contain the latest information on the course.

General Information:

Instructor:	Thomas Wanner
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Fax:	(703) 993-1491
Office hours:	T 2pm-3pm, R 11am-12pm

Lectures:	TR 4:30-5:45pm, Exploratory Hall 4106
Prerequisites:	A thorough knowledge of advanced calculus and linear algebra is assumed. Also, familiarity with the "Theorem-Proof" style of presentation is important.
Textbook:	There is no required textbook for this course.

Important Links:

- Detailed syllabus (including recommended books)
- Homework assignments
- Relevant official GMU policies

Syllabus:

The course introduces basic concepts and techniques of linear functional analysis. These techniques constitute the abstract mathematical framework for solving a variety of applied problems, and applications of the theory will be given throughout the course. In addition to a brief introduction to the necessary topological concepts, the course covers basic Banach and Hilbert space theory, the theory of bounded linear operators between such spaces, as well as the fundamental theorems of functional analysis. A more detailed syllabus can be found here. It will be updated weekly.

Homework Assignments:

Homework problems will be assigned once a week and posted on the homework page as well as on Blackboard. Some of these assignments will be graded and count towards your homework score. While the remaining ones do not have to be handed in, I do advise everyone strongly to study them and write out the solutions properly. I will post detailed solutions as well as videos discussing the solutions on Blackboard, and you will not benefit from this if you have not made a serious attempt at solving the problems.

Grading Policy:

Your final grade in the course will be determined from your performance in a midterm exam, the homework assignments, and a final exam. Weights for these items will be distributed approximately according to the following schedule:

Homework	**Midterm Exam**	**Final Exam**	**Attendance**
50%	20%	20%	10%

The assignment of your course grade is based on the total course score. The following grading scale may serve as a guideline, although changes are possible:

Score above	90%	80%	70%	60%	otherwise
Letter grade	A	B	C	D	F

Thomas Wanner, August 23, 2019.

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The following table contains the schedule for the course. It will be updated regularly throughout the semester. The numbers in the last column indicate the corresponding sections in the book by Kolmogorov and Fomin (see (3) below), which was used to define the preliminary exam syllabus. Additional (and in my opinion, better) book recommendations can be found at the end of this page.

Week	Date		Sections
		I. Introduction	
1	08/27	1. Superposition principle	
	08/29	2. Solvability conditions	
		II. Metric Spaces	
2	09/03	1. Basic definitions	5.1
	09/05	2. Convergence and continuity	5.2, 6.2
3	09/10	3. Topological properties	6.1, 6.4, 6.5, 6.6
	09/12	4. Completeness	7.1, 7.2, 7.4
4	09/17	5. Baire's theorem	7.3
	09/19	6. Separable metric spaces	6.3
5	09/24	7. Completion of a metric space	7.4
	09/26	8. Compactness	10.1, 10.2, 10.3, 10.4
		III. Linear Spaces	
6	10/01	1. Linear spaces	13.1, 13.2, 13.3
	10/03	2. Frechet spaces	17.1, 17.2
		3. Banach spaces	15.1, 15.2
7	10/08	4. Finite-dimensional Banach spaces	
	10/10	5. Compactness of the unit sphere	
		6. Hilbert spaces	16.1, 16.2, 16.8, 16.9
8	**10/15**	**No class! (Columbus Day)**	
	10/17	7. Orthonormal sets	16.3, 16.4, 16.5, 16.6
9	**10/22**	**Midterm Exam**	
	10/24	**Midterm Exam**	
10	10/29	8. Orthogonal projections	16.7
		9. Superposition principle revisited	
		IV. Linear Functionals	
	10/31	1. Continuous linear operators	18.1, 18.2, 22.1, 22.2
11	11/05	2. The Banach algebra L(X,X)	22.3

		3. Dual spaces	19.1, 19.2
	11/07	4. Convexity	14.1, 14.2, 14.3
		5. The Hahn-Banach theorem	14.4, 18.3
12	11/12	6. Reflexive spaces	19.4
		V. Linear Operators	
	11/14	1. Inverse operators	23.1
13	11/19	2. Adjoint operators	23.2, 23.3
	11/21	3. Solvability conditions	
14	11/26	4. Spectrum and resolvent	23.4
	11/28	**No class! (Thanksgiving)**	
15	12/03	5. Completely continuous operators	24.1, 24.2
	12/05	6. The Fredholm-Riesz-Schauder theory	24.3
17	**12/17**	**Final Exam, 4:30-7:15pm**	

The following list contains a number of books that you might find useful for supplementary reading. While the book (3) forms the basis for the preliminary exam schedule, it is rather terse and has not been a favorite of recent students. I personally find (4) a very readable and complete text, but it is unfortunately very expensive. All of these texts cover the basic material of the course. Note that books (6) and (9) are available for free as PDF downloads via the ebook subscription of the library.

1. P. Ciarlet, *Linear and Nonlinear Functional Analysis with Applications*, SIAM, 2013.
2. T. Kato, *Perturbation Theory for Linear Operators*, Springer, 1995.
3. A.N. Kolmogorov and S.V. Fomin, *Introductory Real Analysis*, Dover, 1970.
4. E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley, 1978.
5. P.D. Lax, *Functional Analysis*, Wiley, 2002.
6. B.D. MacCluer, *Elementary Functional Analysis*, Springer, 2009.
7. G.K. Pedersen, *Analysis Now*, Springer, 1989.
8. F. Riesz, B. Sz.-Nagy, *Functional Analysis*, Dover, 1990.
9. B.P. Rynne, M.A. Youngson, *Linear Functional Analysis*, Springer, 2008 (2nd edition).
10. K. Yosida, *Functional Analysis*, Springer, 1980 (6th edition).

Thomas Wanner, August 23, 2019.

Homework Assignments

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In the following table, you can see when homework assignments will be handed out. Unless a due date is specified, the assignment does not have to be turned in. You can find the actual assignments in Blackboard under *Assignments*. In Blackboard under *Course Content*, I will post both solutions to the assignments, as well as videos discussing the solutions in detail. For any additional questions, please see me during office hours.

Homework with a due date will be graded and will count towards your homework score. I will accept homework for grading even after the due date, but only until I have finished grading the already obtained assignments or posted the solutions. This will usually be no earlier than the Monday after the due date. However, if you decide to turn in the homework after the due date, you are doing so at your own risk. After the due date, it is your responsibility to get the assignment to me before I have finished grading. Leaving the assignment in my mailbox or sliding it under the office door will not suffice. No extensions beyond the above policy will be granted.

This table will be updated regularly during the semester.

Week	No.	Assigned on	Due on
1 (08/26-08/30)	0	08/29	---

Thomas Wanner, August 11, 2019.

Relevant George Mason
Official University Policies

The following policies apply to all courses at George Mason University:

1. It is expected that each student will conduct himself or herself within the guidelines of the Honor Code. All academic work should be done with the level of honesty and integrity that this University demands.
2. You are responsible for the accuracy of your own schedule. Check Patriot Web regularly to verify that you are registered for the classes that you think you are. A student who is not registered may not continue to attend class. Faculty are not permitted to grade work of students who do not appear on the official class roster.
3. You are responsible for knowing the last days to drop and add this class.
4. Once the add and drop deadlines have passed, instructors do not have the authority to approve any requests from students to add or drop/withdraw late. It is NOT permissible to drop the class and leave it at that. It needs approval. Late adds (up until the last day of classes) must be reviewed and approved by the department chair of the course being offered. They will be approved only in the case of a documented university error (such as a problem with Financial Aid being processed). All student requests for withdrawals and retroactive adds (after the last day of classes) must be reviewed by the student's academic dean. In the case of students whose major is in COS, this is the office of Undergraduate Academic Affairs in Enterprise.
5. Instructors are required to give the final exam at the time and place published in the Schedule of Classes, as set by the Registrar. It cannot be changed. You need to plan vacation (make plane reservations, etc.) around these published dates.
6. Once final grades have been recorded, instructors cannot accept any work to change that course grade. Grade changes can only be approved when they are due to a calculation or recording error on the part of the instructor.
7. An IN (incomplete) grade is a very special grade that can only be applied for in writing. It can only be given in cases in which a student is passing a course and has a very limited amount of work left to complete the course.
8. Federal law (a law known as FERPA) requires the protection of privacy of student information. Therefore, no instructor on campus can speak about a student's record with anyone other than the student. The record includes how a student is doing in a course, whether a student has attended class, information about grades, whether a paper has been turned in. Anything. This prohibition includes parents, siblings, and spouses, anyone.
