George Mason University
Department of Mathematical Sciences

Rings, fields, and Galois theory

Spring 2020

Course: MATH-721 section 001.

Total Credits: 3.

Purpose: This course is a natural continuation of MATH-621 on groups, rings and field. The main purpose of this course is a solid coverage of field theory and then Galois theory. The hope is that after this course it should be clear that a good knowledge in field theory and Galois theory is good for the working mathematician in general, in particular those concentrating in algebra, combinatorics, discrete geometry and even in graph theory. At the end of the course we will hopefully have covered enough topics and/or examples for the attendees to be able to read advanced monographs and papers on applications of field and Galois theory.

Galois theory is by many considered on of the most beautiful and self-contained algebraic theories for a variety of reasons:

1. It provides explicit tools for the first impossibility results in mathematics, for example, that neither the cube root of two[1] nor the real number π can be constructed by means of a ruler and a compass.
2. The general quintic polynomial equation cannot be solved by radicals.
3. Further, it can be used to describe exactly when a polynomial equation can be solved by radicals (the crowning achievement of Galois!)

These applications are, however, by no means the only ones. A wide range of generalizations are used in current algebraic geometry and number theory as well, although we will not discuss these here.

Prerequisites: Most important is to have mathematical maturity and an open mind. Roughly what I expect is the following:

- Required – Math 621, Algebra (First year graduate course), or equivalent.

Times and Places: TR 12 noon – 1:15 pm. Exploratory Hall, room 4106

Period: From January 21. to May 13.

Dates to keep in mind:

January 28:	Last day to add classes.
February 5:	Last day to drop with 100% tuition refund.
February 11:	Last day to drop (with 50% tuition refund).

Professor:
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Office-hours: TR 3:20 – 4:20 pm, or by appointment.

[1] *Doubling the cube* or the *Delian problem*

Course Text: The following texts will be used in this course:

- (Required) Joseph Rotman, *Galois theory*, Second edition. Universitext. Springer-Verlag, New York, (1998).
- (Recommended) Thomas W. Hungerford, *Algebra*, Graduate Texts in Mathematics (GTM 73) Springer-Verlag, New York-Berlin (1974).

Material: (i) Some background material from chapters III (Rings) and V (Fields and Galois Theory) in Hungerford, and (ii) a good chunk of the book Galois theory by Rotman.

Homework: Homework will be assigned every three weeks or so, a total of four assignments during the semester.

WebSite: All announcements, notes and pdf file handouts for this course will be posted on the following class WebSite:
`http://math.gmu.edu/~geir/courses/721spring20/`

Examinations: There will be no exams in this course.

Grading: Your grade for this course will be based on participation and homework.

Geir Agnarsson
January 21, 2020