

Math 741: Lie Groups

Course Syllabus

Prerequisites

Students must have taken Math 621 and Math 631. It is recommended that you have taken Math 740 (Differential Topology).

Logistics

We will meet electronically 3:00-4:15 on Mondays/Wednesdays. Class will occur by Zoom, at the link:

<https://gmu.zoom.us/j/97060960378?pwd=WUdlV2xVUUUpSYThEeVdDcmhTaWhhZz09>

The meeting ID is 970 6096 0378 and the passcode is 777777

Office hours are Wednesdays 2-2:50pm and Mondays before class, when requested in advance. Other times are available upon request! Office hours will be held on our *Discord server*, or by Zoom in my “personal meeting room”, depending on my success with Discord.

Reach me by email: rgoldin@gmu.edu. If I don't respond within 24 hours, please email me again!

Textbook

The textbook is Brian Hall's “Lie Groups, Lie Algebras, and Representations.” You may also want a copy of Warner's book, “Foundations of Differentiable Manifolds and Lie Groups,” which has a stronger focus on the differential and topological aspects of Lie groups.

Course Content

We will cover some portions of Hall's book more carefully than other parts, but you can expect to cover something in each of Chapters 1-9 and Chapter 11. Time permitting, we will discuss some of Chapters 10 and 12.

We will pull in material from a few other sources. Hall's book focusses on matrix Lie groups, which avoids some of the issues that come up more broadly. I will be providing you with some more general notions during lectures.

Topics we will cover (not necessarily in this order):

- Matrix Lie Groups and Lie subgroups
- Topology of some matrix Lie groups
- The covering relationship of $SU(2) \rightarrow SO(3)$
- Lie groups more generally
- Coverings of Lie groups

- The exponential map for matrix Lie groups
- The exponential map for Lie groups
- Lie algebras
- Homomorphisms of Lie groups and Lie algebras
- Simply connected Lie groups
- The Adjoint representation
- Relationships between Lie groups and Lie algebras
- Root systems, Dynkin diagrams and classical Lie groups
- Classification of simple Lie algebras

Grading

Problem sets 50%

Problem set presentations 20%

Final Exam 30%

Course Structure

I will be lecturing and/or providing problem sets throughout the course. Starting in Week 2, two or three students (you!) will each present a solution to an assigned problem, and answer questions/get feedback from the rest of the group. To give a presentation, you should have a latex-ed version of the relevant problem ready to go, and you can share your screen and talk us through your solution. Please turn on your camera for your presentation. Each student will do this 4-6 times during the semester, depending on how many students enroll in the class and how efficiently the presentations go.

Problem sets will also be submitted for assessment via Gradescope. Late problem sets are accepted up to 1 week late for 75% credit, and subsequently not accepted except under extraordinary circumstances.

Problem sets are graded, however you are encouraged to work with other students on the problem sets (no other resources, just other students!). You must write up your own work individually and without copying from another student's work. Please cite with whom you worked on the top of your first page.

Disability statement

If you are a student with a disability and you need academic accommodations, please see me and contact the Office of Disability Resources at 703.993.2474. All academic accommodations must be arranged through that office.

Honor Code

The University Honor Code is to be followed at all times. Sharing information of any kind about the content of the final exam is prohibited. Plagiarizing from other students/sources or looking up solutions online is prohibited. Any violations will be sent to the Honor Committee and will result in a grade of zero in addition to overall grade reductions/penalties. See the University Honor Code.