

Generalized moment method: time-inversion for parabolic equations in unbounded domains

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We propose a generalized version of the moment method [1] that addresses the initial source identification problem for the heat equation from knowledge of its solution at a time T . Our extension of the method allows us to go beyond the simple case of the heat equation and treat a large class of parabolic advection-diffusion equations of the form:

$$\begin{cases} \partial_t u(x, t) - \mathcal{L}u(x, t) = 0, \\ u(x, 0) = u_0(x), \end{cases}$$

where $\mathcal{L}$ is an elliptic differential operator. The core idea of this new approach is to replace the monomials used in the original method by a family of solutions to elliptic problems associated to the adjoint elliptic operator $\mathcal{L}^*$. These solutions are computed numerically.

Just as in the original method, the initial condition is retrieved as a sum of point sources (Dirac measures) by solving a convex optimization problem in the space of Radon measures. We also propose an extension of the method for the case where $\mathcal{L}$ is endowed with time-dependent coefficients. We hope to be able to generalize this idea to nonlocal parabolic equations featuring operators in the likes of the fractional Laplacian.

References

1. Liu, K. and Zuazua, E. (2026). Moments, time-inversion and source identification for the heat equation. *Inverse Problems*, 42(1), 015009.