On locally finite ordered rooted trees and their rooted subtrees

Aleyah Dawkins, George Mason University, Fairfax, VA – 22030

Abstract

In this talk, we investigate the number of ordered rooted trees of height at most $h \in \mathbb{N}$ of a certain type. We first revisit and generalize a known result about a doubly exponential sequence that describes the number of k-ary ordered rooted trees of height h where $k \geq 2$ is a fixed integer. We then turn our attention to a more general setting for sequences that describe the number of ordered rooted trees of a given height. This has applications in algorithm analyses where searches in rooted trees are performed. We consider infinite ordered rooted trees in which each ordered rooted subtree induced by all the vertices on levels h or less is a finite ordered rooted tree of height h of a certain type. In particular, we study those infinite trees $\mathbf{T}$ in which each vertex has infinitely many descendants.

Finally, we investigate some theoretical properties of the number of ordered rooted subtrees of height at most h for those infinite trees $\mathbf{T}$ that have a finite width when viewed as partially ordered sets (posets) with the root as the sole maximum element. In particular, we show that for large enough h the number of ordered rooted subtrees of height at most h is given by a polynomial in h for h sufficiently large and we determine the degree and the leading coefficient of this polynomial.

Keywords: ordered rooted tree, doubly exponential sequence, infinite ordered rooted tree. the width of a poset.