Analysis Seminar

Schramm-Loewner Evolutions: Past, Present, and Future

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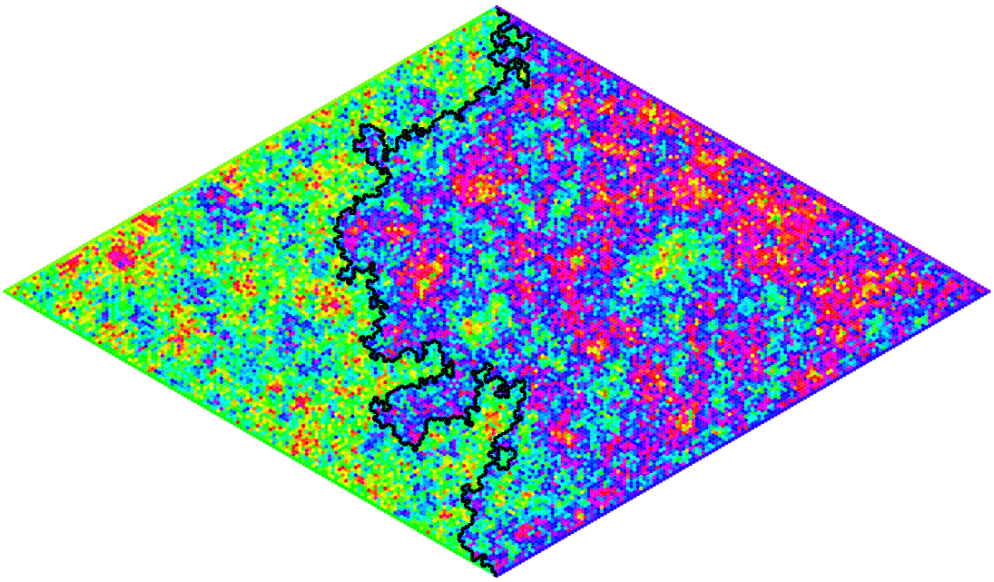

In 1916, Ludwig Bieberbach conjectured that every normalized univalent function on the unit disk,

$$f(z) = z + \sum_{n \geq 2} a_n z^n,$$

satisfies $|a_n| \leq n$ for all n. This led Charles Loewner to introduce a complex differential equation for evolving conformal maps and prove $|a_3| \leq 3$, the first nontrivial case of the conjecture. This work ultimately resulted in Louis de Branges' proof of the conjecture in 1985.

A new and unexpected branch of this story began in the 1980s and 1990s. Physicists and mathematicians conjectured that interfaces in critical planar statistical physics models are conformally invariant and satisfy a spatial Markov property and more importantly have a scaling limit. Although physicists estimated properties of this limit using conformal field theory, there was no mathematical description of these random curves.

In 2000, Oded Schramm realized that any such scaling limit must be characterized by a **stochastic Loewner differential equation**, now called **Schramm-Loewner Evolution (SLE)**. Schramm's work revolutionized modern probability and conformal field theory. The primary focus of this talk is the construction of SLE and the modern developments of the theory, including **multiple SLE** and **conformal loop ensemble (CLE).**

Friday, September 4th
Exploratory 4106
11:30 - 12:30