Evaluation Codes: A Flexible Construction for (Almost) Everything in Coding Theory

Eduardo Camps Moreno, Virginia Tech, Blacksburg, VA – 24061

Abstract

Let $\mathbb{F}_q$ be a finite field and $C \subseteq \mathbb{F}_q^n$ a linear code. The *minimum distance* of C is the smallest number of nonzero entries in a nonzero codeword; if this value is d and $\dim_{\mathbb{F}_q}(C) = k$, we say that C is an $[n, k, d]_q$ code. Constructing good codes means designing subspaces C with prescribed parameters, typically maximizing d for a given dimension. – In this talk, we discuss one of the most versatile frameworks for building such codes: *evaluation codes from monomials*. These are obtained by evaluating a chosen set of monomials at a finite set of points, producing a rich family of linear codes whose properties reflect the algebraic and geometric features of the underlying data. We will see how the combinatorics of the exponent set and the geometry of the evaluation set influence parameters such as the minimum distance, dual code, and other invariants. Classical examples, including Reed-Muller and toric codes, illustrate how this perspective naturally connects coding theory with ideas from commutative algebra and algebraic geometry.

Keywords: finite field, linear code, monomials, toric codes.