

# Cubulating the associahedron

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## Abstract

The associahedron  $K_n$  is a convex polytope capturing the combinatorics of Catalan numbers and whose 1-skeleton can be oriented to give the Hasse diagram of the Tamari lattice. The "cubical realization" of the Tamari lattice by bracket vectors is a particularly nice way of mapping Catalan objects to points on the boundary of an axis-aligned parallelepiped. I will discuss a Minkowski decomposition of the associahedron into Bruhat interval polytopes, and a mixed subdivision of the associahedron into  $(n - 1)!$  (combinatorial) cubes which is closely related to the cubic realization of the Tamari lattice. This subdivision arises from a degeneration of the toric variety of the associahedron into a union of products of smooth toric Richardson varieties.

**Keywords:** Tamari lattice, Catalan object, Bruhat interval polytope, toric Richardson variety.