

# Minkowski rings of polytopes and power closed ideals

*Geir Agnarsson*, George Mason University, Fairfax, VA – 22030

## Abstract

The *Minkowski ring* of a collection  $\mathcal{P}$  of convex polytopes in a Euclidean space is the ring generated by the indicator functions  $\{[P] : P \in \mathcal{P}\}$  where the addition is given naturally as the sum of the functions and where the multiplication is determined by  $[P] \cdot [Q] = [P + Q]$ ; the indicator function of the Minkowski sum of the polytopes  $P$  and  $Q$ . If  $\mathcal{P}$  contains  $d$  polytopes, then such rings, when viewed as algebras over the complex number field, can then be presented as  $\mathbb{C}[x_1, \dots, x_d]/I$  where  $I$  is the ideal describing all the relations implied by the polytopes in  $\mathcal{P}$ . Among many nice properties that such ideals  $I$  have is that they are *power closed* in the sense that if  $f(x_1, \dots, x_d) \in I$  then  $f(x_1^i, \dots, x_d^i) \in I$  for every natural number  $i$ . In this talk we discuss power closedness of ideals and related closure operators and their properties. – This is joint work with Jim Lawrence.

**Keywords:** Polytopes, Minkowski sum, Minkowski ring, power closed ideals.