ISOMETRIES OF BANACH SPACES OF ANALYTIC FUNCTIONS

FLAVIA COLONNA

In a general sense, an *isometry* is a transformation that preserves distance between elements. Examples in the Euclidean plane are rotations, translations, and reflections. In operator theory, the "Isometry Problem" is:

Identify the isometries of a Banach space.

In 1932, Banach proved that given a compact metric space K, the surjective isometries T of the linear space $C(K)$ of continuous real-valued functions have the characterized as follows.

$$Tf(t) = h(t)f(\varphi(t)) \quad \text{for } f \in C(K), t \in K,$$

where $|h(t)| = 1$ and φ is a homeomorphism of K.

Many similar results emerged afterwards on many classical Banach spaces, including the Hardy spaces and the Bergman spaces, which then prompted the study of the *weighted composition operators* (WCO), i.e. linear operators of the form

$$Tf = u(f \circ \varphi),$$

for some functions u and φ, called the *symbols* of the operator T.

In this talk, an overview of the history will be given with focus on Banach spaces of analytic functions on the open unit disk in $\mathbb{C}$. We shall then highlight the similarity that arises among some classes of Banach spaces among the composition operators or WCOs that are isometries, as well as exceptional case of a space of analytic functions known as the *Bloch space*.

Some recent results and open questions in both one and several variables will conclude the talk.