Abstract: Physicists’ ability to measure and control quantum matter has grown from manipulating single atoms in the 1990s to coherently controlling thousands of them, frequently atom-by-atom. This had led to deep insights about interacting quantum matter, and new phases of matter and dynamical phenomena. I will discuss our recent progress advancing this frontier, in which we (theorists) have partnered with experimentalists to create programmable matter in both real space and in so-called synthetic dimensions, and to understand the many-body physics that results. One approach uses optical potentials in real space to program arbitrary lattice geometries for fermions, allowing experiments to perform “quantum simulations” of models and regimes that have never before been accessible. Another, perhaps more surprising, approach uses synthetic dimensions, systems with degrees of freedom that behave in a way that can mimic motion in space. I will focus on Rydberg atom synthetic dimensions, describing recent experiments that observed topological edge states and how interactions can lead to novel phases of matter, including quantum strings and membranes, and to parastatistical quasiparticles, a type of particle beyond fermions and bosons (and beyond anyons).