Statistics of Attractor Embeddings in Reservoir Computing

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A recent branch of Artificial Intelligence (AI) or Neural Networks that can handle time-varying signals often in real time has emerged as a new direction for signal analysis. These dynamical systems are usually referred to as reservoir computers (RC). These RCs can be driven by signals from very complex external systems and often can reconstruct signals from those drive systems that are not in the original signal sent to the RC. How can a system reconstruct a signal it hasn't seen from a signal from a different part of the input systems? The answer to this involves some deep mathematical results from differential geometry and dynamical systems. But explaining the main requirement is actually rather simple and straightforward. A central question in the operation of these systems is whether a reservoir computer (RC) when driven by only one time series from a driving or source system is internally recreating all the drive dynamics. This leads to an important question of when we have data from such a setup, but no real mathematical model how can we test this? That's something experimentalists and engineers who might want to use this AI setup would face. For example, for RCs constructed from actual physical systems like interacting lasers or analog circuits, the RC dynamics may not be known well or at all. I present some simple statistics that can help test for mathematical relationships between a drive system and the RC by using the time series from both systems. One statistic is called the continuity statistic and it is modeled on the mathematical definition of a continuous function that everyone encounters in their first calculus class. We can use these statistics to show evidence for complex functions from a drive system to the reservoir network and vice versa. So in designing and using such AI systems we can now test to see if they are behaving as we want them to and giving us the right answers.