# Math 301-DL1 Number Theory 

George Mason University

Fall 2021 Syllabus

## Course Description

Learning Objectives: To understand and make use of the concepts of prime numbers, congruences, multiplicative functions, and cryptology, and to develop general mathematical skills.

Prerequisite: Completion of 6 hours of MATH.
Textbook: Rosen, Elementary Number Theory and its Applications, 6th Edition, Pearson, 2010
Course Modalities: All online, half asynchronous, and half synchronous.

## Course Meetings:

- Synchronous classes: Thursdays, 1:30-2:45pm, Zoom
- Asynchronous learning and course home page: Blackboard, https://mymasonportal. gmu.edu/


## Instructor Information

Name: Dr. Kirsch (she/her/hers)
Student Office Hours: Tuesdays 1:30-2:45pm, or by appointment, on Zoom
Email Address: rkirsch4 @ gmu.edu

## Grading and Course Requirements

The grading rubric for each problem is
3 Demonstrates full achievement of the learning objective
2 Demonstrates significant progress toward the learning objective
1 Demonstrates some progress toward the learning objective
0 Does not demonstrate progress toward the learning objective
Letter grades in the course will be determined by the percentage of points you have earned and the following table.

| $\mathrm{A}+$ | A | $\mathrm{A}-$ | $\mathrm{B}+$ | B | $\mathrm{B}-$ | $\mathrm{C}+$ | C | $\mathrm{C}-$ | D | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $97-100$ | $93-96$ | $90-92$ | $87-89$ | $83-86$ | $80-82$ | $77-79$ | $73-76$ | $70-72$ | $60-69$ | $0-59$ |

Course components and point values: Subject to change, but only to your advantage.
Syllabus and Course Policies Quiz 6
Community and Collaboration Portfolio 18
Preparation, Participation, and Practice Portfolio 18
Reflection Homework 14
Chapters 1, 3:
Weeks 1-4 Quizzes 24
Exam $1 \quad 24$
Chapters 4, 6:
Weeks 6-9 Quizzes 24
Exam $2 \quad 24$
Chapters 7, 8:
Weeks 11-14 Quizzes 24
Exam $3 \quad 24$
Total points in course
200
Retake opportunities: You may complete at most one retake problem each week to replace your score on a quiz or exam problem corresponding to the same learning objective.

## Course Calendar

| Week | Course Topics | Relevant Problems |
| :---: | :---: | :---: |
| Week 1 <br> August 24-30 | 1.3 Mathematical Induction 1.5 Divisibility | $\begin{aligned} & 1.3 \# 2,3,4,6,7,8,9,10,12,13, \\ & 14,20,21,24,26,27,36 \\ & 1.5 \# 1-22,26,27,28,30,31,36-43 \end{aligned}$ |
| Week 2 August 31 - September 5 Labor Day September 6 | 3.1 Prime Numbers <br> 3.2 The Distribution of Primes <br> 3.3 Greatest Common Divisors and their Properties | $\begin{aligned} & 3.1 \# 1,3,6,7,8,9,12,14,15,16 \text {, } \\ & 17,18,19,26,27,28 \\ & 3.2 \# 1,3,4,5,6,7,8,9,12,13 \text {, } \\ & 14,18 \\ & 3.3 \# 1,2,3,4,5,6,7,8,9,10,11 \text {, } \\ & 12,13,14,17,18,19,20,25,27,31 \end{aligned}$ |
| Week 3 <br> September 7-13 | 3.4 The Euclidean Algorithm <br> 3.5 The Fundamental Theorem of Arithmetic, Proof | 3.4 \#1-10, 14 <br> 3.5 Understand and be able to explain each step of the proof. |
| Week 4 September 14-20 | 3.5 The Fundamental Theorem of Arithmetic, Applications <br> 3.6 Factorization Methods and Fermat Numbers <br> 3.7 Linear Diophantine Equations | $\begin{aligned} & 3.5 \# 1-5,6,7,8,9,10,11,28,29 \text {, } \\ & 30,31,34,35,36 \\ & 3.6 \# 1-4 \\ & 3.7 \# 1-10 \text { odd, } 13-18 \text { odd } \end{aligned}$ |
| Week 5 September 21-27 | Chapter 1: The Integers and Chapter 3: Primes and Greatest Common Divisors Catch-Up, Review, and Exam 1 | See Weeks 1-4 |
| Week 6 September 28-October 4 | 4.1 Introduction to Congruences <br> 4.2 Linear Congruences | $\begin{aligned} & 4.1 \# 1-4,6,7,8,9,10-14,16,17 \text {, } \\ & 20-23,25,27,29,31,32,34,41,42 \\ & 4.2 \# 1-15 \text { odd, } 19 \end{aligned}$ |
| Week 7 <br> October 5-10 | 4.3 Sun Zi's Remainder Theorem 6.1.1 Wilson's Theorem | $\begin{aligned} & 4.3 \# 1-12,16,17,20,21 \\ & 6.1 \# 1-22 \text { odd, } 27,29,34,41 \end{aligned}$ |
| Week 8 October 13-18 | 6.1.2 Fermat's Little Theorem | 6.1 \#1-22 odd, 27, 29, 34, 41 |
| Week 9 October 19-25 | 6.2 Pseudoprimes 6.3 Euler's Theorem | $\begin{aligned} & \hline 6.2 \# 1-19 \text { odd } \\ & 6.3 \# 1-12 \text { odd, } 17,19 \end{aligned}$ |
| Week 10 <br> October <br> November 1 | Chapter 4: Congruences and Chapter 6: Some Special Congruences Catch-Up, Review, and Exam 2 | See Weeks 6-9 |
| Week 11 November 2-8 | 7.1 The Euler Phi-Function <br> 7.2 The Sum and Number of Divisors | $\begin{aligned} & 7.1 \# 1-8,12,13,14,17,19 \\ & 7.2 \# 1-4,7-14,20,21 \end{aligned}$ |
| Week 12 <br> November 9-15 | 7.3 Perfect Numbers and Mersenne Primes 7.4 Mobius Inversion | $\begin{aligned} & 7.3 \# 1-14,15 \\ & 7.4 \# 1-6,10,13,14,19,21 \end{aligned}$ |
| Week 13 <br> November 16-22 | 8.1 Character Ciphers <br> 8.3 Exponentiation Ciphers | $\begin{aligned} & 8.1 \# 1-14 \\ & 8.3 \# 1-6 \end{aligned}$ |
| Week 14 <br> November 23, 29 | 8.4 Public Key Cryptography | 8.4 \#1-8, 11-15 |
| Week 15 <br> November <br> December 5 | Chapter 7: Multiplicative Functions and Chapter 8: Cryptology Catch-Up and Review | See Weeks 11-14 |
| December 8-13 | Exam 3 | See Weeks 11-14 |

