MATH 322

ADVANCED LINEAR ALGEBRA

PROF. ANTON LUKYANENKO - SUMMER 2023

Class:	MTWR 10:30 – 12:35 in 4010 Horizon	Dates:	May 22 – June 22 (5 weeks)
Office hours:	MTWR 1:00 – 2:00 in 4113 Exploratory	Email:	alukyane@gmu.edu

Overview

Linear algebra is an essential tool throughout theoretical and applied mathematics.

Having taken Math203, you've seen the essentials of linear algebra and how it is used.
With Math300, you are ready to look at how and why it works – and learn advanced methods.

1. Essentials review: vectors, matrices, dot products, cross products, inverses, transposes.
2. Broader perspective:
   a. Freeing linear algebra from specific choices of scalars, coordinates and matrices.
   b. Fields, which generalize the concept of "scalar", including $\mathbb{R}$, $\mathbb{C}$, and beyond.
   c. Vector spaces, including $\mathbb{R}^d$ and the set $\mathbb{R}[x]$ of all polynomials.
   d. Coordinate systems, using a set of *basis vectors*.
   e. Transformations between vector spaces, including coordinate changes.
3. Everyday tools:
   a. Determinants and invertibility, for undoing a transformation, if possible.
   b. Eigenvalues, eigenvectors, giving directions of stretching and rotation.
   c. Applications to systems of linear equations.
   d. Gram-Schmidt orthonormalizaton, for getting really nice coordinate systems.
   e. Matlab, for actually working with linear algebra.
4. Advanced methods:
   a. Advanced vector space constructions, including quotients, duals, sums, tensors.
   b. Normal forms (RCF, JCF), for choosing the best basis for any transformation.
   c. Spectral theorem, for diagonalizing transformations that preserve dot products.
   d. Perron-Frobenius, for finding positive eigenvectors and making Google work.
   e. Principal axis theorem, for analyzing data and finding eigenfaces.

Course components

- Reading: We will largely follow *Linear Algebra: An Introductory Approach* by Curtis (link).
- Lectures: There will be *some* lectures as needed. Pay attention and ask lots of questions.
- Groupwork: We will *primarily* work in groups during class, with worksheets for guidance.
  - Groups will change every worksheet. Get to know your group.
  - Stick together to power through the definitions and questions.
  - Make sure everyone knows what's going on: learn to listen and assist.
  - I'll come around and answer questions from each *group* as needed.
- Presentations: I will ask you to present some of your ideas to the class.
  - Present to your classmates, not to me.
  - Pick a "nemesis team" that will evaluate your response.
  - Answer any questions from the nemesis team and other classmates.
  - Listen actively (even when presenting!) and ask questions.
- Written reports: Once a worksheet is completed, you will write a brief report on it.
  - The audience is **you**. Write down the key ideas you need to remember, including key definitions, proofs, examples, and any relevant code.
  - Aim for about 2 standard LaTeX pages per report.
  - Typed reports are ideal but not required, and the exact formatting is up to you.
  - Reports will be graded (with up to 2 resubmissions), with comments, as:
    0 = "unsubmitted/incomplete" 1= "redo" 2="good" 3="great"
  - Reports are due by the start of the next class after a worksheet is finished or a graded report is returned. Submitted reports will be graded by the next day.
- Idea assessments: Interview-style oral exams with a 5-minute time limit. No notes.
  - This is a chance to convince me one-on-one you know the content.
  - For each of the 4 units, I will announce ~10 questions ahead of time. Study them.
  - During an assessment, I will pick 3 questions at random. Answer 2 questions to pass.
  - Assessments can be repeated with no penalty, but units have to be passed in order.
  - We will have five assessments, each Thursday, and one on a class day of your choice.
  - Assessments will take place during office hours (unless you need a different time).
- Collaboration: Work together, study together, but submit your own work.
  Any copied/plagiarized content will result in an "incomplete" grade and a warning on the first occurrence, and an official honor code violation on any further occurrences.
  Of course, technical terms should not be altered. When in doubt, ask me.
- Participation and collegiality: Active, professional, participation is required every day.
  After 3 warnings, each additional violation will result in a 1/3 letter grade reduction.
  Of course, extreme cases will be resolved/reported more quickly.

Grades

Grades will be assigned as follows, up to effects of the participation clause above.
The goal is to incentivize you to both study all of the content and learn most of it very well.

- A+: pass all 4 units, and a grade of "great" on every report.
- A: pass all 4 units, and a grade of "great" on all but 3 reports, which are at least "good".
- B: pass 3 units, and a grade of at least "good" on all reports.
- C: pass 2 units with at least 4 attempts, and a grade of at least "good" on all but 3 reports.
- D: pass 2 units, with a grade of at least "good" on 2/3 of the reports.
- F: anything below the above standards.

Absences, Accommodations, and Exceptions

Missed classes, arriving late, and lack of participation disturb the workflow of your team and will result in warnings and penalties based on the "participation and collegiality" clause.
Missed idea assessments will not be rescheduled.

That said, I'm happy to be accommodating within reason, including all legally-required accommodations, so please ask for exceptions that you think are warranted.

Resources

- General Advising: http://advising.gmu.edu/
- Student Support and Advocacy Center: https://ssac.gmu.edu/
- Disability Services: http://ds.gmu.edu/
- Counseling and Psychological Services: https://caps.gmu.edu/
- Compliance Diversity and Ethics Office: https://diversity.gmu.edu/

Want more math?

Having fun? Get involved! Ask me about any of the following:

- Mason Experimental Geometry Lab (MEGL): do math research in the fall, for credit.
- Graduate school: study math for 6 years and become a pro in your field.
- Learning assistant program: help me teach Math322 in the fall.
- Have random math questions? Feel free to ask, or share any cool math you're learning.

Disclaimers

This syllabus may be updated as necessary to improve the class. Reports graded as "great" may still contain mistakes. Idea assessments will be recorded for quality assurance purposes.