Modern Applied Mathematics I

Math 413-001

Fall 2021

This is the web page **http://math.cos.gmu.edu/~wanner/courses/m413f21/index.html**. It will be updated regularly and always contain the latest information on the course.

General Information:

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Office hours:	M 4pm-5pm, W 11am-12pm, and by appointment

Lectures:	MW 5:55pm-7:10pm, Exploratory Hall L111
Prerequisites:	Grade of C or better in Math 203 and 216 (or 214).
Textbook:	There is no required textbook for the course, I will post handwritten lecture notes on Blackboard after every class. While I will draw the material from a variety of sources, the following two texts can be used for supplementary reading: • Mark H. Holmes: *Introduction to the Foundations of Applied Mathematics* (Springer, 2019). • J. David Logan: *Applied Mathematics* (Wiley, 2013). You can download the pdf version of the first book using the library's Springer ebook subscription.

Important Links:

- Detailed syllabus
- Relevant official GMU policies
- Homework assignments will be posted on Gradescope.

Syllabus:

This course provides a basic introduction into important techniques of modern applied mathematics. We will discuss the interplay between discrete and continuous mathematics, as well as the role of computational mathematics. Particular focus will be put on revealing mathematical structure from equilibrium models in discrete and continuous systems. We will cover dimensional analysis, perturbation methods, stability and bifurcations, kinetics, and diffusion. A more detailed syllabus can be found here. It will be updated weekly.

Homework Assignments:

Homework problems will be assigned once a week and posted on Gradescope. These assignments will be graded and count towards your homework score. At the end of the semester, I will drop some of the lowest scores. More details will be announced later in the semester.

Exams:

The midterm exam is Wednesday, October 13, 5:55-7:10pm, the final exam is Monday, December 13, 4:30-7:15pm.

Grading Policy:

Your final grade in the course will be determined from your performance in the homework assignments, a midterm exam, the final exam, and your attendance and class participation. Weights for these items will be distributed approximately according to the following schedule:

Homework	Midterm Exam	Final Exam	Participation
40%	25%	25%	10%

The assignment of your course grade is based on the total course score. The following grading scale may serve as a guideline, although changes are possible:

Score above	90%	80%	70%	60%	otherwise
Letter grade	A-, A, or A+	B-, B, or B+	C or C+	D	F

Modern Applied Mathematics I

Math 413-002

Fall 2021

The following table contains the schedule for the course. This page will be updated regularly throughout the semester.

Week	Date		Notes Pages	Book Sections
		I. Dimensional Analysis and Scaling		
1	08/23	1. Basic Simplification	1-6	[H] 1.1, [LS] 6.1, L 1.1.1
	08/25	2. Conditioning and Sensitivity	7-11	[LS] 6.1
2	08/30	3. Dimensional Methods	12-17	[H] 1.2, [LS] 6.2, [L] 1.1.2
	09/01	4. The Buckingham Pi Theorem	18-24	[H] 1.3, [L] 1.1.3, 1.1.4
3	09/08	5. Scaling	25-32	[LS] 6.3, [L] 1.2.1, 1.2.3
		II. Perturbation Methods		
4	09/13	1. Formal Approximations for Root Finding	33-37	[H] 2.2.1, 2.2.2, [M] 1.2, 1.3
	09/15	2. Expansions via Computer Algebra	38-40	[H] 2.2.2
		3. The Implicit Function Theorem	41-43	
5	09/20	4. Justification and Error Estimates	44-48	[M] 1.4
	09/22	5. The Newton Polygon	49-56	[M] 1.5
6	09/27	6. Rescaled Coordinates	57-61	[H] 2.3, 2.4, [M] 1.6
	09/29	7. Bifurcations	62-68	[L] 1.3.2
7	**10/04**	**Review**		
		III. Perturbations of Differential Equations		
	10/06	1. Perturbations of Second-Order Linear Equations	69-71	[M] 2.1
		2. Regular Perturbations of Initial Value Problems	72-76	[H] 2.2.3, [L] 3.1.1, [M] 2.4
8	10/12	3. Regular Perturbations of Boundary Value Problems	77-79	[M] 2.5
	10/13	**Midterm Exam, 5:55-7:10pm**		
9	10/18	4. Oscillatory Problems and Secular Terms	80-83	[L] 3.1.2, [M] 4.1
	10/20	5. Poincare-Lindstedt Expansions	84-90	[L] 3.1.3, [M] 4.2
10	10/25	6. Boundary Layer Analysis	91-94	[H] 2.5, [L] 3.2.3, 3.3.1
	10/27	7. Matched Asymptotic Expansions	95-103	[H] 2.6, [L] 3.3.2, 3.3.3, 3.3.4
		IV. Stability and Bifurcations		
11	11/01	1. Qualitative Study of Dynamical Systems		
		2. Dynamics of Scalar Flows		
		3. Effects of Parameter Variation		
		4. Equilibrium Stability in Higher Dimensions		
		5. Case Study: The Tacoma Narrows Bridge		

V. Kinetics and Diffusion

1. Law of Mass Action
2. Conservation Laws
3. Michaelis-Menten Law
4. Diffusion

17 **12/13** **Final Exam, 4:30-7:15pm**

For the course, I will draw material from the following books:

- M.H. Holmes: *Introduction to the Foundations of Applied Mathematics*, Springer, 2019. [H]
- C.C. Lin, L.A. Segel: *Mathematics Applied to Deterministic Problems in the Natural Sciences*, SIAM, 1988. [LS]
- J.D. Logan: *Applied Mathematics*, Wiley, 2013. [L]
- J.A. Murdock: *Perturbations*, SIAM, 1999. [M]

Thomas Wanner, October 20, 2021.

Relevant George Mason
Official University Policies

The following policies apply to all courses at George Mason University:

1. All students taking courses with a face-to-face component are required to follow the university's public health and safety precautions and procedures outlined on the university Safe Return to Campus webpage. Similarly, all students in face-to-face and hybrid courses must also complete the Mason COVID Health Check daily, seven days a week. The COVID Health Check system uses a color code system and students will receive either a Green, Yellow, or Red email response. **Only students who receive a "green" notification are permitted to attend courses with a face-to-face component.** If you suspect that you are sick or have been directed to self-isolate, please quarantine or get testing. Faculty are allowed to ask you to show them that you have received a Green e-mail and are thereby permitted to be in class.
2. Students are required to follow Mason's current policy about facemask-wearing. As of August 11, 2021, **all community members are required to wear a facemask in all indoor settings, including classrooms**. An appropriate facemask must cover your nose and mouth at all times in our classroom. If this policy changes, you will be informed; however, students who prefer to wear masks either temporarily or consistently will always be welcome in the classroom.
3. It is expected that each student will conduct himself or herself within the guidelines of the Honor Code. All academic work should be done with the level of honesty and integrity that this University demands.
4. You are responsible for the accuracy of your own schedule. Check Patriot Web regularly to verify that you are registered for the classes that you think you are. A student who is not registered may not continue to attend class. Faculty are not permitted to grade work of students who do not appear on the official class roster.
5. You are responsible for knowing the last days to drop and add this class.
6. Once the add and drop deadlines have passed, instructors do not have the authority to approve any requests from students to add or drop/withdraw late. It is NOT permissible to drop the class and leave it at that. It needs approval. Late adds (up until the last day of classes) must be reviewed and approved by the department chair of the course being offered. They will be approved only in the case of a documented university error (such as a problem with Financial Aid being processed). All student requests for withdrawals and retroactive adds (after the last day of classes) must be reviewed by the student's academic dean. In the case of students whose major is in COS, this is the office of Undergraduate Academic Affairs in Enterprise.
7. Instructors are required to give the final exam at the time and place published in the Schedule of Classes, as set by the Registrar. It cannot be changed. You need to plan vacation (make plane reservations, etc.) around these published dates.
8. Once final grades have been recorded, instructors cannot accept any work to change that course grade. Grade changes can only be approved when they are due to a calculation or recording error on the part of the instructor.
9. An IN (incomplete) grade is a very special grade that can only be applied for in writing. It can only be given in cases in which a student is passing a course and has a very limited amount of work left to complete the course.
10. Federal law (a law known as FERPA) requires the protection of privacy of student information.

    Therefore, no instructor on campus can speak about a student's record with anyone other than the student. The record includes how a student is doing in a course, whether a student has attended class, information about grades, whether a paper has been turned in. Anything. This prohibition includes parents, siblings, and spouses, anyone.
