MATH 464/664 Fall 2022

Professor: Ben Schweinhart **E-Mail:** bschwei@gmu.edu

Lectures: 15:00 – 16:15 PM on Tuesdays and Thursdays in EXPL 4106.

Office Hours: 16:30 – 17:30 PM Tuesdays and 14:00-15:00 Thursdays in EXPL 2212.

Communication: Important course announcements will be posted on Blackboard and/or by email. The best way to reach me is by email.

Course Description: Linear algebra is almost ubiquitous in both pure mathematics and data science. The objectives of the course are twofold: to cover linear algebra from the more abstract viewpoint necessary for its applications in pure mathematics, and to provide hands-on experience with computational methods. The course will cover abstract concepts such as dual spaces, matrix decompositions such as Jordan canonical form and SVD, normed linear spaces, inner product spaces, and Perron–Frobenius Theory. Computational methods will include principal component analysis, dimensionality reduction, and the PageRank algorithm.

Prerequisites: CS112 and MATH322, equivalent courses, or permission of the instructor. The course assumes knowledge of the basics of linear algebra (e.g. vector spaces, bases, and eigenvalues and eigenvectors), some familiarity with programming, and the ability to write basic mathematical proofs.

References: The textbook for the course is Linear Algebra in Action (Second Edition) by Harry Dym.

Grading: 40% Homework, 40% Computational Projects, 20% final presentation.

Homework Homework will be assigned on a bi-weekly basis. Only a subset of questions will be graded. The lowest homework grade will be dropped, and the rest will be weighted equally.

Computational Projects Several computational projects will be assigned during the semester. More details will be given in class.

Presentations Students will prepare a 30 minute presentation on a topic of their choosing for the last week of the semeester. More details including a list of topics will be given later in the semester. Topics will include further applications of linear algebra to data science and to pure mathematics. Students will be graded on effort and on their understanding of the material. Before presenting, they should meet with the instructor to discuss the material and their presentation.

Tentative Weekly Schedule (book sections in parentheses)

1. (1.1-1.3) Review of basic linear algebra (vector spaces, linear transformations, bases).
2. (1.4-1.5,4.1) Dual spaces and transposes, isomorphisms and coordinates, subspaces and triangular matrices.

3. (2.1-2.3) Dual space isomorphism, the matrix algebra, Gaussian elimination.
4. (2.4-2.8) Conservation of dimension, matrices and dimension, regression and bases.
5. (4.2-4.6) Matrix decompositions (overview), invariant subspaces, diagonalizability.
6. (4.7-4.11) Generalized eigenvectors, Jordan chains, and Jordan canonical form.
7. (4.12-4.13) Symmetry and the singular value decomposition (SVD), uniqueness caveats, implications for PCA.
8. (9.1,10.1) Metrics and inner products and multi-dimensional scaling (MDS), determinant, trace, and norm.
9. (5.1-5.5) Matrix norms, operator norms, kernel PCA and kernel regression.
10. (7.1-7.3,8.1-8.2,11.1-11.2) Inner products, quadratic forms, regularization and ridge regression/kernel ridge

11–14. Special topics, which could include Perron–Frobenius theory and Markov chains, dynamical systems, and linear programming.

15. Student presentations.

Gender identity and pronoun use: If you wish, please share your name and gender pronouns with me and how best to address you in class and via email. I use "he/him" for myself and you may address me as "Ben" or "Dr./Prof. Schweinhart" in email and verbally.

Campus Closure or Emergency Class Cancelation/Adjustment Policy: If the campus closes, or if a class meeting needs to be canceled or adjusted due to weather or other concern, students should check Blackboard for updates on how to continue learning and for information about any changes to events or assignments.

Academic Integrity: Mason is an Honor Code university; please see the Office for Academic Integrity for a full description of the code and the honor committee process.

Disability Accomodations: Disability Services at George Mason University is committed to providing equitable access to learning opportunities for all students by upholding the laws that ensure equal treatment of people with disabilities. If you are seeking accommodations for this class, please first visit http://ds.gmu.edu/ for detailed information about the Disability Services registration process. Then please discuss your approved accommodations with me.

Disclaimer: This syllabus is a general plan for the course and is subject to change at the instructor's discretion.