Math 465 : Mathematics of Data Science

- **Prerequisites:** MATH 214, MATH 464 or permission of the instructor. The course assumes a solid background of basic linear algebra (vector spaces, bases, matrices, eigenvalues) and ability to write basic mathematical proofs. Projects will require basic programming/computational experience.
- **Recommended prerequisites:** MATH 352 or STAT 350/356/360.
- **Instructor:** Tyrus Berry, tberry@gmu.edu, http://math.gmu.edu/~berry/
- **Office:** Blackboard Collaborate
- **Office hours (EST):** Mondays 2pm-3pm, and 6pm-7pm, and Fridays 1pm-2pm, and by appointment.
- **Course Website:** Blackboard, https://mymasonportal.gmu.edu/
- **Book:** Notes
- **Classroom:** Asynchronous online

Overview

This course is aimed at providing rigorous mathematical foundation for data science related methodologies often omitted in computational curriculum. Matrix algebra, differential equations and graph theory techniques are integrated throughout the course. Foundational aspects of data science including mathematical theory of linear and nonlinear dimension reduction, elements of spectral graph theory, parametric and non-parametric modeling and data-driven dynamics identification and discovery are introduced. The following specific concepts will be discussed:

1. Types of data: continuous vs. categorical, meta-structures (time, space, network)
2. Overfitting: Dimensionality reduction vs. regularization
3. Modeling/Regressions: Parametric vs. Nonparametric
4. Dimensionality/Features: Intrinsic vs. extrinsic
5. Dynamical vs. static models: identification of model type and learning dynamics from data

Approximate Weekly Schedule

- Weeks 1-2: Linear dimensionality reduction
  - PCA/MDS, Sard's Theorem
- Weeks 2-3: Nonlinear dimensionality reduction
  - ISOMAP, Intro to Manifolds and geodesics, Kernel PCA, Mercer's Theorem, Diffusion maps
  - tSNE/UMAP, autoencoders, gradient based methods
- Week 4: Spectral methods and elements of graph theory, information entropy, random graph models.
- Week 5-6: Kernel regression and Reproducing Kernel Hilbert Spaces (RKHS)
  - Positive definite kernels, Moore-Aronszajn Theorem
- Weeks 7-8: Introduction to Riemannian Geometry
  - Riemannian metric, normal coordinates, Laplace-Beltrami operator, curvature

- Weeks 9-10: Manifold Learning theory
  - Proof that certain graph Laplacians converge to Laplace-Beltrami operators on manifolds
- Week 11: Applications of Manifold Learning
  - Representing diffeomorphisms and operators, the diffusion forecast
- Weeks 12-13: Persistent homology
  - Homology of simplicial complexes, persistence diagrams for homology and computation
- Week 14: Dynamics and data: discovery of dynamics, dynamic mode decomposition, sparse dynamics identification.

Office Hours

Office hours will all be held online via Blackboard Collaborate

Student Engagement: Weekly Learning Plan

Your week should be split up into three study periods (on three different days) each consisting of:

1. Viewing a short lecture video on Blackboard.
2. Reading a section or two in the book.
3. Working a quick "quiz" problem on Blackboard (unlimited attempts).

The remaining time will be devoted to projects. Projects will have a large computational aspect and Matlab/Octave/FreeMat are highly recommended but alternatives may be acceptable, please discuss in advance if you would like to use an alternative.

Grading

Grades will be based on two components:

1. 60% - Computational projects
2. 40% - Blackboard quizzes

Academic Policies

Mason is an Honor Code university; please see the Office for Academic Integrity for a full description of the code and the honor committee process. With collaborative work, names of all the participants should appear on the work. Collaborative projects may be divided up so that individual group members complete portions of the whole, provided that group members take sufficient steps to ensure that the pieces conceptually fit together in the end product. Other projects are designed to be undertaken independently. In the latter case, you may discuss your ideas with others and conference with peers on drafts of the work; however, it is not appropriate to give your paper to someone else to revise. You are responsible for making certain that there is no question that the work you hand in is your own. If only your name appears on an assignment, your professor has the right to expect that you have done the work yourself, fully and independently.

Disability Accommodations

Disability Services at George Mason University is committed to upholding the letter and spirit of the laws that ensure equal treatment of people with disabilities. Under the administration of University Life, Disability Services implements and coordinates reasonable accommodations and disability-related services that afford equal access to university programs and activities. Students can begin the registration process with Disability

Services at any time during their enrollment at George Mason University. If you are seeking accommodations, please visit http://ds.gmu.edu/ for detailed information about the Disability Services registration process. Disability Services is located in Student Union Building I (SUB I), Suite 2500. Email: ods@gmu.edu
Phone: (703) 993-2474

Non-Discrimination Policy

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The University is dedicated to ensuring access, fairness and equity for minorities, women, individuals with disabilities, and veterans (as covered by law) in its educational programs, related activities and employment. George Mason University shall thus maintain a continuing affirmative action program to identify and eliminate discriminatory practices in every phase of university operations.

Any employee who becomes aware of sexual harassment or other potentially discriminatory behavior must contact Compliance, Diversity, and Ethics.

Retaliation against an individual who has raised claims of illegal discrimination or has cooperated with an investigation of such claims is prohibited.