Complex Analysis: MATH 661 GMU

Fall 2022

Instructor:	Dr. Mahamadi Warma	**Time:**	**MW: 1:30 PM–2:45 PM**
Email:	mwarma@gmu.edu	**Place:**	**Exploratory Hall 4106**

Office Hours: MW 9:30 AM–10:30 AM or by appointment.

Office: Exploratory Hall, room 4461.

Phone: 703-993-1488.

Text Book: John B. Conway. **Functions of one complex variable I, Springer-Verlag, 2nd Edition**.

Course Description. Differentiation and integration of complex functions. Infinite series. Analytic functions. Analytic continuation. Multiple-valued functions. Conformal mapping.

Objectives. The students will acquire knowledge of the fundamental notions in the theory of Complex Analysis and to be able to solve some problems involving the following mathematical concepts and techniques:

1. Mapping of the planar regions onto the planar regions using fractional bilinear transformations.
2. Application of the Cauchy integral formula to prove the Fundamental Theorem of Algebra.
3. Application of the Residue Theorem to compute improper real integrals.
4. The Cauchy Integral Theorem.
5. Powers series.
6. Laurent series. Singularities.
7. The Residue Theorem.
8. The Argument Principle.
9. The Maximum Modulus Theorem.
10. Characterizations of analytic functions of a complex variable.

Syllabus and Time Distribution.

1. **The Complex number systems (6 hours)**
   - 1.1 The real numbers.
   - 1.2 The field of complex numbers.
   - 1.3 The complex plane.
   - 1.4 Polar representation and roots of complex numbers.
   - 1.5 The extended plane and its spherical representation (stereographic projection).
2. **Metric spaces and the topology of $\mathbb{C}$ (9 hours)**
   - 2.1 Definition and examples of metric spaces.

2.2 Connectedness.

2.3 Sequences and completeness.

2.4 Compactness.

2.5 Continuity.

2.6 Uniform convergence.

3. **Elementary properties and examples of analytic functions (6 hours)**

3.1 Power series.

3.2 Analytic functions.

3.3 Möbius transformations.

4. **Complex integration (9 hours)**

4.1 Riemann-Stieltjes integrals.

4.2 Power series representation of analytic functions.

4.3 Zeros of an analytic function.

4.4 The index of a closed curve.

4.5 Cauchy theorem and Integral formula.

4.6 The homotopic version of Cauchy's theorem.

4.7 The open mapping theorem.

4.8 Goursat's theorem.

5. **Singularities (9 hours)**

5.1 Classification of singularities.

5.2 Residues.

5.3 The argument principle.

6. **The maximum modulus theorem (6 hours)**

6.1 The maximum principle.

6.2 Schwarz's lemma.

Homework: Problems will be assigned regularly throughout the semester. Students are expected to solve all the assigned problems, and some of these problems will be handed in and graded. Students are allowed to discuss assigned problems with classmates, but solutions should be written individually.

Tests: There will be one (1) final exam. The test is closed-book and closed-notes.

Grading Policy: The graded homework for 70% and the final exam for 30%. Equivalence between scores and letters, recommended by GMU, is given in the table below:

A+	A	A-	B+	B	B-	C+	C	C-	D	F
>97	>93	>90	>87	>83	>80	>77	>73	>70	>60	60-0

Other useful books:

1. Lars Ahlfors. **Complex Analysis** 3^{rd} **edition. JMcGraw-Hill**.

2. E. M. Stein and R. Shakarchi. **Complex variables. Princeton Lectures in Analysis II**.

Attendance and Course Policy:

- Students are expected to participate in all classes. In the event that you must miss a class, you are responsible to ask classmates concerning announcements made and the material discussed.
- Makeup exams are only possible with an acceptable excuse. Examples of such excuses are religious holy days, family emergencies, school sponsored events, job interviews, or sickness. All absences require documentation. Notify me of any religious holy days within the first 2 weeks of the semester. Changing the date of the final exam for unusual circumstances, or because three or more finals are scheduled in one day, requires the approval from the professor at least a week prior to the last day of classes. If absence from the final exam is unexcused, the grade for the course is F.

Cellular Phones in the Classroom: Students must turn off all cellular phones and other communication devices when in the classroom. Emergency personnel should notify the professor at the beginning of the course and set phones to vibrate mode.

GMU Policies: The University Catalog, http://catalog.gmu.edu, is the central resource for university policies in university academic affairs. Further policies are available at http://universitypolicy.gmu.edu/ All members of the university community are responsible for knowing and following established policies.

Honor Code: Students are expected to follow the honor code https://oai.gmu.edu/mason-honor-code/ Lack of knowledge of the honor code is not a reasonable excuse for its violation.

Disability Services: Reasonable accommodations are available for students who have a documented disability. Please contact Disability Services if you require accommodations: Office of Disability Services, Student Union Building I (SUB I), Room 4205, Phone: 703-993-2474.