Partial Differential Equations

Math 678-001

Fall 2022

This is the web page **http://math.gmu.edu/~wanner/courses/m678f22/index.html**
It will be updated regularly and always contain the latest information on the course.

General Information:

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Office hours:	T 3-4pm, R 1-2pm, and by appointment

Lectures:	R 7:20-10:00pm, Exploratory Hall 4106
Prerequisites:	A thorough knowledge of multi-variable calculus and elementary differential equations is assumed. Also, familiarity with the theorem-proof style of presentation is important.
Textbook:	There is no required textbook for this course.

Important Links:

- Detailed syllabus (including recommended books)
- Relevant official GMU policies

Lecture notes, solutions to homework assignments, reading assignments, and additional materials can be found on the Blackboard site for this course. Homework assignments can be found on Gradescope, which is linked through Blackboard as well. Please make sure to check there regularly!

Syllabus:

This course provides an introduction to partial differential equations. Specific topics include maximum principles, basic Sobolev space theory, linear elliptic equations and regularity, as well as conservation laws and shocks. It will also be demonstrated how these methods can be used in applications. A more detailed syllabus can be found here. It will be updated weekly.

Homework Assignments:

Homework problems will be assigned once a week and posted on Gradescope, which you can access through the left sidebar in Blackboard. Most of these assignments will be graded and count towards your homework score. While the remaining ones do not have to be handed in, I do advise everyone strongly to study them and write out the solutions properly. I will go through many of the homework problems in the following class and you will not benefit from this if you have not made a serious attempt at solving them.

Grading Policy:

Your final grade in the course will be determined from your performance in the homework assignments, a final project/presentation, and your attendance and class participation. Weights for these items will be distributed approximately according to the following schedule:

Homework	Final Project/Presentation	Attendance
60%	30%	10%

Thomas Wanner, August 22, 2022.

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The following table contains a preliminary schedule for the course. This page will be updated regularly throughout the semester.

Week	Date		Notes, Additional Readings
1	08/25	**I. Introduction**	
		1. What is a PDE?	[SW] 1.1
		2. The Transport Equation	[SW] 1.2.3
		3. The Laplace Equation	[SW] 1.2.2, 2.3.2
2	09/01	4. The Heat Equation	[SW] 1.2.1, 2.3.3
		5. The Wave Equation	[SW] 1.2.4, 2.3.1
3	09/08	6. Classification of PDEs	[SW] 1.3.1, 1.3.2, 1.3.3
		II. Conservation Laws and Shocks	
		1. Basic Definitions and Examples	[RR] 3.1, 3.2
		2. Blowup of Smooth Solutions	[RR] 3.3
4	09/15	3. Weak Solutions	[RR] 3.4.1, [E] 3.4.3
		4. Multiplicity	[RR] 3.4.2
5	09/22	5. The Lax Shock Condition	[RR] 3.4.3, 3.5, [E] 3.4.1
		6. Other Conditions and Results	[RR] 3.6, [E] 3.4.1, 3.4.4
		III. Maximum Principles	
		1. The Weak Maximum Principle	[RR] 4.1.1, [E] 6.4.1
6	09/29	2. The Strong Maximum Principle	[RR] 4.1.2, [E] 6.4.2
7	10/06	3. A Priori Bounds	[RR] 4.1.3
		4. An Existence Proof for the Dirichlet Problem	[RR] 4.2
		5. Symmetry of Positive Solutions	[RR] 4.3
		6. Parabolic Equations	[RR] 4.4
8	**10/13**	**No class! Time to work on projects!**	

9	10/20	7. Decay of the Zero Number **IV. Sobolev Spaces** 1. Review of Lebesgue Integration 2. The Banach Spaces L^p	
10	10/27	3. Weak Derivatives 4. Sobolev Spaces 5. Compact Embeddings	
11	11/03	6. Sobolev Embedding Theorem 7. Boundary Values 8. Poincare Inequality **V. Linear Elliptic Equations** 1. Elliptic Differential Operators 2. Solution Types for the Dirichlet Problem	
12	11/10	**Project Presentations** 3. The Lax-Milgram Lemma 4. Garding's Inequality 5. Existence of Weak Solutions 6. Fredholm Theory 7. Regularity and Classical Solutions	Robert Dudzinski
13	11/17	**Project Presentations**	Biniyam Tibebu, Shraddha Verma, Julia Seay, Iffat Sarfraz
14	**11/24**	**No class! (Thanksgiving Break)**	
15	12/01	**Project Presentations**	Cheng-Kang Chao, Joseph Franks, Alonso Ogueda Oliva, Madeline Horton, Matthew Kearney
16	12/08	**Project Presentations**	Martha Hartt, Kirsten Tate, Aman D'Souza, Jessica Masterson

While there is no assigned textbook for the course, I draw material mostly from the following books:

- R.A. Adams, J.J.F. Fournier: *Sobolev Spaces*, 2nd edition, Academic Press, 2003.
- L.C. Evans: *Partial Differential Equations*, 2nd edition, American Mathematical Society, 2010.
- D. Gilbarg, N.S. Trudinger: *Elliptic Partial Differential Equations of Second Order*, Springer, 2001.
- R. Haberman: *Mathematical Models*, SIAM, 1998.
- M. Renardy, R.C. Rogers: *An Introduction to Partial Differential Equations*, 2nd edition, Springer, 2004.
- E. Sander, T. Wanner: *Theory and Numerics of Partial Differential Equations*, book draft, 2022.

Relevant George Mason
Official University Policies

The following policies apply to all courses at George Mason University:

1. It is expected that each student will conduct himself or herself within the guidelines of the Honor Code. All academic work should be done with the level of honesty and integrity that this University demands.
2. You are responsible for the accuracy of your own schedule. Check Patriot Web regularly to verify that you are registered for the classes that you think you are. A student who is not registered may not continue to attend class. Faculty are not permitted to grade work of students who do not appear on the official class roster.
3. You are responsible for knowing the last days to drop and add this class.
4. Once the add and drop deadlines have passed, instructors do not have the authority to approve any requests from students to add or drop/withdraw late. It is NOT permissible to drop the class and leave it at that. It needs approval. Late adds (up until the last day of classes) must be reviewed and approved by the department chair of the course being offered. They will be approved only in the case of a documented university error (such as a problem with Financial Aid being processed). All student requests for withdrawals and retroactive adds (after the last day of classes) must be reviewed by the student's academic dean. In the case of students whose major is in COS, this is the office of Undergraduate Academic Affairs in Enterprise.
5. Instructors are required to give the final exam at the time and place published in the Schedule of Classes, as set by the Registrar. It cannot be changed. You need to plan vacation (make plane reservations, etc.) around these published dates.
6. Once final grades have been recorded, instructors cannot accept any work to change that course grade. Grade changes can only be approved when they are due to a calculation or recording error on the part of the instructor.
7. An IN (incomplete) grade is a very special grade that can only be applied for in writing. It can only be given in cases in which a student is passing a course and has a very limited amount of work left to complete the course.
8. Federal law (a law known as FERPA) requires the protection of privacy of student information. Therefore, no instructor on campus can speak about a student's record with anyone other than the student. The record includes how a student is doing in a course, whether a student has attended class, information about grades, whether a paper has been turned in. Anything. This prohibition includes parents, siblings, and spouses, anyone.
