

Math 680: Industrial Mathematics

Fall 2022: W 7:20pm – 10:00pm, Exploratory Hall 4106

Instructor: Dr. Daniel Anderson

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Office Hours: Online To Be Determined, and by appointment.

Texts: *Notes for Industrial Mathematics*, D.M. Anderson. Other useful (not required) references *Mathematical Models in the Applied Sciences*, A.C. Fowler (Cambridge University Press, 1997); *Advanced Mathematical Methods for Scientists and Engineers*, C.M. Bender and S.A. Orszag (McGraw-Hill, 1978); *Perturbation Methods*, E.J. Hinch

Prerequisites: Familiarity with ordinary and partial differential equations. Interest in scientific applications.

Course Goals: To develop mathematical techniques that can be used to solve problems arising in the physical sciences and/or in industrial settings. First, this course will introduce asymptotic and perturbation methods for differential equations. Second, the course will focus on applied problems selected mainly from the broad areas of heat transfer, fluid mechanics, and solidification. No prior knowledge of or experience with these application areas will be assumed. A goal will be to develop skills required to formulate a sound mathematical model whose solution will provide answers and insight into the original problem.

Topics:

Asymptotic and Perturbation Methods:	Landau order symbols, asymptotic and convergent series, regular and singular perturbation methods, matched asymptotic expansions, multi-scale methods
Heat Transfer in Solids and Liquids:	conservation laws, heat equation, similarity solutions, homogenization theory
Introduction to Fluid Mechanics:	Navier–Stokes Equations, boundary conditions fluid statics, thin film approximation, scaling analysis, viscous gravity currents
Solidification of a Pure Materials and Alloys:	The Stefan problem, free boundary problems, linear stability analysis
Additional Topics:	Porous Media/Filtration, Double-diffusive Convection, Mushy Layers, Melting Problems, ...

Grading Policy: The course grade will be based on homework (60%) and a project (40%).

Project: This will involve the application of the mathematical methods developed in the class to a particular application area related to those discussed in class. Each student will present a preliminary plan for their project near the middle of the semester and a final version at the end of the semester.

Honor Code: It is expected that each student in this class will conduct himself or herself within the guidelines of the Honor Code. All academic work should be done with the level of honesty and integrity that this University demands.

Math 680 lectures – planned Fall 2022 - subject to change

Date (Sect.)	homework	Topic
Week 1		introduction to modeling, variety of topics; order notation and asymptotic concepts; regular perturbation of an algebraic equation
Week 2		Mass on a spring: nondimensionalization, scaling, break down of regular perturbations introduce matched asymptotic expansions,
Week 3		Mass on a spring: matched asymptotic expansions, Multi-scale methods
Week 4		Multi-scale methods
Week 5		Heat Transfer Problems: conservation law, heat equation, similarity solution in 1D, homogenization theory – rapidly layered media
Week 6		Introduction to Fluid Mechanics: Navier–Stokes Equations, Interfacial boundary conditions
Week 7		Fluid Statics Gravity and Surface Tension
Week 8		Viscous Gravity Current; Lubrication Approximation, scaling analysis, reduction to PDE, similarity solution, comparison with data
Week 9		Midterm Presentations
Week 9		Gravity Current in Porous Media: scaling analysis, reduction of problem to single PDE, similarity solution, predictions
Week 10		Thin-Film stuff (tear films, etc.),
Week 11		Solidification of a Pure Material: Stefan problem, (free boundary problem), scaling, nondimensionalization, similarity solutions and other solutions.
Week 12		Solidification of an Alloy: Similarity solutions with/without mushy layers
Week 13		Linear Stability Analyses in Solidification
Week 14		Final Presentations