George Mason University
Department of Mathematical Sciences

Combinatorial Structures: Graph Enumerations

Spring 2022

Course: MATH-723, Combinatorial Structures, section 001.

Total Credits: 3.

Purpose: This course will server as an introduction to graph enumeration, or enumerative graph theory. This is a topic which is quite natural to start to cover right after the definition of a graph, but in most introductory books and courses on graph theory is left out manly for the following reasons: (i) There often are more practical, "useful" and "urgent" topics one needs to cover. (ii) Many times one needs to introduce helpful tools from algebra and/or analysis in order to interpret functional results in a combinatorial manner. (iii) Finally, many of the results in graph enumeration are exact and conceived from "old-school" formal mathematics. As such, many of the main theorems were developed decades ago, and even earlier. – All this does not mean that their value has diminished in any way, on the contrary: they often serve as theoretical incarnation of the ultimate verification of empirical result generated heuristically from computers.

This course could have been called "*A second course in graph theory*" or "*Graph Theory II*", except for the fact that we will not assume too much from MATH-641, the first course on graph theory. The bottom line, we will introduce many new topics not seen in a first graph theory course without assuming too much from it. As a result, the course can be viewed as a self-contained continuation of a first course in graph theory.

Prerequisites: Most important is to have mathematical maturity and an open mind. Roughly what I expect is listed as follows:

- Required – Math 621, Algebra (First year graduate course), or equivalent.
- Preferred – (1) Math 325, Discrete Mathematics II, or equivalent. (2) Math 203, Matrix Algebra (Linear Algebra), or equivalent.

NOTE! In the GMU catalog online at `https://catalog.gmu.edu/` in "Find a Course" when typing "MATH 723", there are no required prerequisites listed as this is left up to the instructor.

Times and Places: MW 3:00 – 4:15 pm. Exploratory Hall 4106.

Period: From January 24. to May 18.

Dates to keep in mind:

January 31:	Last day to add classes
February 7:	Last day to drop (100% tuition refund)
February 14:	Last day to drop (50% tuition refund)
March 1:	Last day to drop (no tuition refund)

Professor:

Geir Agnarsson
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Office-hours: MW 12:20 – 1:20 pm, or by appointment.

Course Text: There is no designated text necessary for this course. However, we will use the following texts as reference. Results assumed known that are stated without formal proofs will be from this following book. It is recommended to have it available:

- Thomas W. Hungerford *Algebra*, Graduate Texts in Mathematics (GTM-73), Springer Verlag, New York (1974), 12th printing (2003) or most recent one.

The lectures will be self contained. Most of the material will be selected from the following books:

1. Geir Agnarsson; Raymond Greenlaw *Graph Theory, Modeling, Applications, and Algorithms*, Pearson Prentice Hall, (2007).
2. Narsingh Deo, *Graph Theory*, Prentice Hall, (1974).
3. Frank Harary; Edgar M. Palmer, *Graphical Enumeration*, Academic Press, New York-London, (1973).
4. Richard P. Stanley, *Enumerative Combinatorics, Volumes I and II*, Cambridge University Press (1997) and (1999) respectively.

Material: Selected topics on the following: (i) Labeled enumeration of graphs. (ii) Unlabeled enumeration of graphs: (iii) Groups and graphs, (iv) Pólya's enumeration theorem, (v) Rooted and unrooted trees. (vi) Power group enumeration theorem. (vii) Unlabeled blocks (i.e. non-separable graphs). (viii) Asymptotics (if time allows).

Homework: Homework will be assigned sporadically and not on a regular basis. Expect about $4 - 5$ assignments during the semester, about once every couple of weeks.

BlackBoard: All announcements, notes and pdf file handouts for this course will be posted on the course BlackBoard site.

Examinations: There will be no exams in this course.

Grading: Your grade for this course will be based on participation, that is, attendance and homework.

Geir Agnarsson
January 23, 2022