When is the group of units as additively closed as possible?

Neil Epstein, George Mason University, Fairfax, VA – 22030

Abstract

A ring is *unit-additive* if a sum of units is always either a unit or nilpotent. For example, $k[x]$ and $k[x]/(x^2)$ are unit-additive, but $\mathbb{Z}$ is not. In fact, an affine semigroup ring $R[M]$ is unit-additive if and only if R is unit-additive and M has no nontrivial invertible elements. In the algebraic variety setting, we get a characterization of unit additivity in terms of the nonexistence of nonconstant maps to the punctured affine line, somewhat reminiscent of Liouville's theorem from complex analysis. The concept of unit additivity leads to the related concept of unit-additivity dimension – i.e. how far is an integral domain from being unit-additive? It turns out that rings of unit-additivity dimension 1 are of some interest, as they include the rings of integers of number fields, all power series rings, and most local rings. We construct rings of all unit-additivity dimensions and show that in the affine setting, unit-additivity dimension is bounded above by Krull dimension. We also construct the *unit-additive closure* of an integral domain D, being the smallest subring of the fraction field of D that is unit-additive, as a localization at a certain multiplicative set in D. This is joint work with Jay Shapiro.

Keywords: unit, nilpotent, integral domain, local ring, Krull dimension, unit-additive ring.