FALL 2020

Mathematical Methods in Physics. 3 credits.

CRN: 81984 **Duration**: Aug 24, 2020 - Dec 16, 2020

Location: Planetary Hall 220

Time: 10:30 am - 1:10 pm, Friday

Instructor: Igor Mazin, imazin2@gmu.edu. Online office hours by arrangement, please send me an email. I usually respond within a few hours.

Text: *Mathematical Methods in the Physical Sciences by Mary Boas, any edition, Chs. 2, 3, 6, 8, 10, 11 (partially), 12 (partially), 14.* In Addition, secondary source will be used for individual sections. In particular, for section 1, Qualitative Methods in Quantum Theory by A.B. Migdal, any edition, Ch. 1.1, for section 5-7, as well as various handouts. A helpful addition to Mary Boas' book, which is less verbose and somewhat more advance is *Mathematical Methods for Physicists. A Comprehensive Guide, any edition, by George Arfken et al.*

Course philosophy: This is not a math class. Proofs of theorems will be discussed on practical level, not with mathematical rigor. The emphasize will be on practical skills useful for analyzing mathematical expression, taking integrals, solving differential equations, working with vectors and matrices, etc.

- **Class format**
  - The class will be conducted face to face (the assigned room allows for double distancing of >10 feet apart)
  - The class will be largely in the "flipped classroom" format, that is, the students are expected to have read the assigned literature, including handouts, and work on the suggested problems. The amount of material will not allow for postponing till the last day, but assumes systematic working during the week.
  - Attendance is mandatory. If you are sick, or cannot attend for any other reason, do not hesitate to contact me ASAP. We will work out some solution (a more detailed written description of your homework or an individual online session). This said, a large part of your learning experience will be collective discussions in the class, so do your best to attend them all.
  - Homework is due the morning on the day of the class. You can use any source, but be aware that getting the right answer will matter less for your grades than demonstrating your work, your reasoning, and your path to the answer.
  - Every homework problem will be discussed in the class. Every students will be expected to participate either by presenting their solution at the whiteboard, or by participation in the discussion. Participation *will* affect your grades.
- **Homework (40% of grade)**
  - Depending on the complexity of the problem, between 3 and 7 problems will be given each week. It is not expected that every student will be able to solve every problem, and failing to carry the calculations to the end on one or two problems will not necessarily reduce you grades, providing you will have demonstrated your work.

  - A large part of the grade will be assigned according to the participation during the class. Asking question counts as participation. Asking a question that the professor cannot answer right away counts especially highly.
  - Working in study groups of 2-3 persons is allowed and encouraged. Any tool for collective work can be utilized. Some recommend Awwapp.com, which is a virtual whiteboard, which can be shared and saved – I have no personal experience with it. I also encourage students to exchange their personal information in the first class in order to facilitate working together. This said, copying homework from other students constitutes cheating and will be treated as violation of the Academic Integrity rules. Because each problem will be discussed in class, copying without understanding will be immediately obvious.
- **Exams and tests.**
  - Midterm exam (30% of grade): 10:30 am - 1:10 pm October 16, 2020.
  - Final exam (30% of grade): 10:30 am - 1:10 pm December 10 or 17 (TBD), 2020.
  - Each exam will contain 4-5 problems on the level of homework assignments.
  - Occasionally short 30 min quizzes will be given in class. Their grades will count together with the homework grades
  - Students can use lecture notes, textbooks and Mathematica/Matlab during the tests. As in the homework, a right answer without proper derivation (which no software will be able to provide) counts as nil.
- **The goal of this course:** This is not a math class. Proofs of theorems and rules will be discussed on the level of physical, not mathematical rigor. Emphasis will be on tools and methods that physicists commonly utilize in their work, and the ultimate goal is to make you comfortable with applying these tools.
- **Students with disabilities:** If you are a student with a disability and need academic accommodations, please see me and contact the Office of Disability Services at 703.993.2474. All academic accommodations must be arranged through that office.
- **GMU Diversity Statement:** GMU does not tolerate racism, sexism, and bigotry, and encourages diversity. The full GMU diversity statement can be read here: http://ctfe.gmu.edu/professional-development/mason-diversity-statement/
- **Academic Integrity:** GMU is an Honor Code university; GMU Honor Code: To promote a stronger sense of mutual responsibility, respect, trust, and fairness among all members of the George Mason University community and with the desire for greater academic and personal achievement, we, the student members of the university community, have set forth this honor code: Student members of the George Mason University community pledge not to cheat, plagiarize, steal, or lie in matters related to academic work. Please see the University Catalog at https://catalog.gmu.edu/policies/honor-code-system/#text for a full description of the code and the honor committee process.
- **Tentative schedule (some variation are possible). At least a month before the course starts you will be given a *non-graded* test to assess your existing skills. It will be due one week before the first class, where the test problems will be discussed. More detailed information about each portion will be made available on the blackboard.**

Week 1: (i) Discussion of the assessment test (ii) Qualitative methods in physics (iii) Less common integration techniques

Wks. 2-3: Complex variables.

- Analyticity.
- Laurent series.
- Poles and residues.
- Complex cuts.

Week 4: Taking integrals with the contour method. Jordan's lemma.

Wks. 5-7: Linear algebra

- matrix formulation of the linear least square problem
- Properties of eigenvectors and eigenvalues. Example: benzene molecule
- Rayleigh-Schrödinger and Dirac-Wigner perturbation theories
- Löwdin perturbation theory. Applications: Crystal field for *p* and *d* orbitals as ligand field. Trigonal distortion.

Week 8: Midterm exam

Week 9: Non-Cartesian coordinate systems. Jacobians. Polar, spherical, affine, parabolic, elliptical coordinates. Example of problems solvable in particular systems.

Week 10: Vector calculus: covariant notation, Kronecker and Levi-Civita symbols. Del operator. Scalar products, cross-products, triple products in covariant notations. Convolution of δ and ε. Reciprocal space and reciprocal lattices.

Week 11: Vector calculus: gradient, divergence, curl. Relation to double cross products and triple products. Operators in different coordinate systems.

Wks. 12-13: . Special functions

- Factorial and Γ functions. Sterling formula. Application to vacancy entropy
- Orthogonal polynomials. Spherical and cubic harmonics. Bessel functions.
- Spherical Bessel functions and spherical vs. plane waves.

Week 14: TBD

Week 15: Final exam