Topology, Algebraic Geometry, & Dynamics Seminar

Plaquette Percolation on the Torus

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Classical percolation theory studies phase transitions marked the emergence of a giant component in random media. Higher dimensional generalizations of these phenomena are of interest in theoretical physics, specifically in the study of lattice gauge theories (discretized models of quantum field theory). We show the existence of topological phase transitions in plaquette percolation on a d-dimensional torus TNd defined by identifying opposite faces of the cube $[0, N]\, d$. The model we consider starts with the complete $(i-1)$-dimensional skeleton of the cubical complex TNd and adds i-dimensional cubical plaquettes independently with probability p. Our main result is that if $d = 2i$ is even and

$$\varphi^* : H_i\left(P; \mathbb{Q}\right) \to H_i\left(TNd; \mathbb{Q}\right)$$

is the map on homology induced by the inclusion $\varphi : P \to TNd$, then

$$\mathbb{P}_p\left(\varphi^* \text{ is nontrivial}\right) \to 0$$

if $p < 1/2$ and

$$\mathbb{P}_p\left(\varphi^* \text{ is nontrivial}\right) \to 1$$

if $p > 1/2$ as $N \to \infty$. We also show that 1-dimensional and $(d-1)$-dimensional plaquette percolation on the torus exhibit similar sharp thresholds at $p\hat{c}$ and $1 - p\hat{c}$ respectively, where $p\hat{c}$ is the critical threshold for bond percolation on $\mathbb{Z}^d$. Time permitting, we will present new work establishing analogous results for Potts lattice gauge theory.

Date: **Friday, April 15, 2022**
Time: **1:30-2:30 pm**
Place: **Exploratory Hall, Room 4208**

For special accommodations, please contact David Carchedi via email at `dcarched@gmu.edu`.