

# On the Partition Regularity of $ax + by = cw^mz^n$

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## Abstract

The study of Ramsey theory is about structures that are partition regular. A Diophantine equation is partition regular if for any finite partition of  $\mathbb{N}$ , one of the cells of the partition contains a solution to the given equation. Schur proved in 1916 using elementary arguments that the equation  $x + y = z$  is partition regular, which is one of the earliest results of this type. It was only in 2010 – 2011 that Bergelson and Hindman independently proved that the equation  $x + y = wz$  is partition regular, and their arguments were much more sophisticated than those of Schur since they used very special types of ultrafilters. In joint work with Richard Wagner, we gave an almost complete classification of those  $a, b, c, m, n \in \mathbb{N}$  for which the equation  $ax + by = cw^mz^n$  is partition regular. In doing so, we also obtained a partial generalization of the Grunwald-Wang Theorem from number theory, which we will also discuss. If time permits, we will discuss the classical  $c_p$ -partitions of Rado and give a sketch of our proof methods.

**Keywords:** Ramsey theory, partition regular, Grunwald-Wang Theorem.