MULTILINEAR AND UNIFORM BOUNDS IN HARMONIC ANALYSIS

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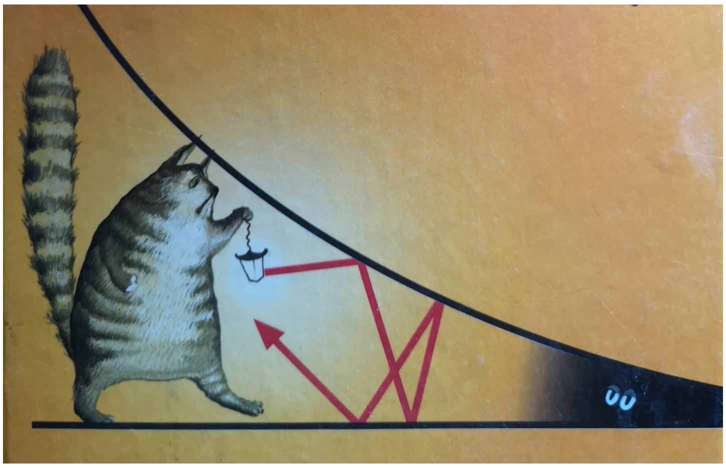

Abstract. Analogously to how multiplication by a matrix expresses a linear operator on a vector space, a linear operator between function spaces can be expressed via integration

$$Tf(x) = \int_{\mathbb{R}^n} K(x, y) f(y) \mathrm{d}y,$$

where the kernel K takes the place of the matrix. Many linear operators on functions occurring naturally in fields of applied and pure science are characterized by singular kernels that are not amenable to standard measure-theoretic arguments. Holomorphic extensions or solutions of the Laplace equation for fixed boundary data and even the identity operator are given by integration against singular kernels. Developed starting from the '60s, Calderón-Zygmund theory provides a vast array of tools for dealing with such operators.

Similarly to how one can approximate smooth functions with Taylor polynomial expansions, one needs to understand *multilinear singular integral operators* to efficiently deal with non-linear or more complicated linear operators on functions. This has been recognized in multiple fields of mathematics: stochastic analysis (Lyons), rough semi-linear PDE (Calderón), ergodic theory (Tao & Green), etc.

A theory that encompasses both linear and multilinear singular integral operators is still in the making. The main difficulty arises from the more complex groups of symmetries multilinear operators tend to possess.

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