

# Combinatorics and geometry of the alternating sign matrix polytope

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## Abstract

The polytope  $ASM_n$ , the convex hull of the  $n \times n$  alternating sign matrices, was introduced by Striker and by Behrend and Knight. We derive many properties of  $ASM_n$  and compare them to those of the polytope  $B_n$ , the convex hull of the  $n \times n$  permutation matrices. For example, we show that a  $d$ -dimensional face of  $ASM_n$  has at most  $2^d$  vertices and  $4(d-1)$  facets, for  $d \geq 2$ . We describe combinatorially the facets of a face and a basis for the subspace parallel to a face. We prove that no face of  $ASM_n$  has the combinatorial type of  $B_3$ .

**Keywords:** convex hull, alternating sign matrix, permutation matrix.