# Combinatorial rings in mixed characteristic

*Olivia Strahan*, University of Michigan, Ann Arbor, MI – 48109

**Abstract**

In this talk, I will introduce a framework for defining new classes of rings analogous to various well-studied combinatorial algebras over a field. Given a discrete valuation ring $(V, t)$, the polynomial ring $V[x_1, \ldots, x_n]$ is $\mathbb{Z}^n$-graded by multidegree in the variables, and up to multiplication by a unit the homogeneous elements are of the form $t^{m_0} x_1^{m_1} \ldots x_n^{m_n}$. I call these elements ***t-monomials***.

I will define $t$-Stanley-Reisner rings, $t$-semigroup rings, and $t$-toric face rings by replacing monomials with $t$-monomials in the definitions of Stanley-Reisner rings, semigroup rings, and toric face rings, respectively. I will also outline a few classical results which I have adapted to these new rings by exploiting the evaluation map $x_0 \mapsto t$, which induces an algebraically well-behaved bijection between $t$-monomials and monomials in one extra variable. These are techniques which I believe are particularly relevant for studying questions which are known in equicharacteristic but still open in mixed characteristic.

**Keywords:** discrete valuation ring, $t$-Stanley-Reisner, $t$-semigroup, $t$-toric face rings.